

Lecture 14: main exercises

Exercise 14.1. Let $Q = 1 \rightarrow 2 \rightarrow 3$.

- (a) Determine the simple, projective, and indecomposable Q -representations.
- (b) Show that the cluster characters of the indecomposable representations are exactly the non-initial cluster variables in A_Q .

Lecture 14: additional exercises

Exercise 14.2. Let C be an ADE Cartan matrix of rank n and let Q be the quiver whose signed adjacency matrix is

$$\begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}$$

in block diagonal form.

- (a) Explain in words what these quivers look like.
- (b) Show that mutating at vertices $1, \dots, n$ in any order (where we label vertices by the rows of the above matrix) results in a quiver which is the same as the original one up to permuting labels of vertices.
- (c) When C is of type A_2 , show that after some sequence of mutations you can obtain quivers with arbitrarily many arrows. In particular, infinitely many different quivers will appear in the exchange graph of A_Q .