Lecture 15: main exercises

Exercise 15.1.

Show that if G is trivial then $\mathrm{R}_{G,N} \cong N_{\mathcal{O}} \times_{N_{\mathcal{K}}} N_{\mathcal{O}}$ and that the convolution product is just the normal tensor product.

Exercise 15.2.

If $G = GL_1$ acting on $N = \mathbb{C}$ with weight w explain why, for $k \in \mathbb{Z}$, the lattice description of $\mathrm{R}_{k\omega_1^\vee}$ is $\{v \in t^{kw}\mathbb{C}_{\mathcal{O}} \cap \mathbb{C}_{\mathcal{O}}\}$.

Exercise 15.3. Let $f : X \times Y$ by a map of schemes and consider the convolution product on $\mathrm{QCoh}(X \times_Y X)$ as discussed in class. If we denote by $\Delta : X \to X \times_Y X$ the diagonal embedding show that $\Delta_* \mathcal{O}_X$ plays the role of the identity with respect to this convolution product.

Exercise 15.4.

Show that there exists a natural embedding $e : N_{\mathcal{O}} \hookrightarrow \mathrm{R}_{G,N}$ and that $e_* \mathcal{O}_{N_{\mathcal{O}}}$ is the identity element for the convolution product.

Exercise 15.5. Fix $G = GL_2$, $N = \mathbb{C}^2$ the standard representation and consider $X := \mathsf{Gr}_{\leq 2\omega_1^\vee - \omega_2^\vee}$.

(a) Show that X has a resolution $\mathsf{Gr}_{GL_2}^{1,-1} = \{L_2 \overset{1}{\supset} L_1 \overset{1}{\subset} L_0 : tL_0 \subset L_1, tL_2 \subset L_1\}$.

(b) Show that L_0 belongs to X.

(c) Show that every other lattice $L \neq L_0$ in X does not contain L_0 nor is it contained in L_0.

(d) Compute the codimension of $L \cap L_0 \subset L_0$ for every $L \in X$ (X has two $G_{\mathcal{O}}$-orbits so you just have to consider one lattice in each orbit).

(e) Conclude that $\mathrm{R}_{G,N}$ is not a vector bundle over $X \subset \mathsf{Gr}_G$.