WARTHOG 2018, Lecture V-1

Main Exercise 1. We assume $\mathbf{G} = \mathrm{SL}_2$ and $\ell > 2$ ($\ell \neq p$).

Assume first that ℓ divides $q-1$.

(a) Determine the irreducible characters with trivial defect.

(b) Let $\theta, \theta' \in \operatorname{Irr} T$ such that θ and θ' take the same values on ℓ'-elements. Show that the irreducible consituents of $R_e(\theta)$ and $R_e(\theta')$ are in the same block.

(c) Given $\theta \in \operatorname{Irr} T$, let

$$e_\theta = \frac{1}{|T_{\ell'}|} \sum_{t \in T_{\ell'}} \theta(t) t^{-1}.$$

(i) Show that $\overline{\mathbb{F}}_\ell[G/U] e_\theta$ is a projective $\overline{\mathbb{F}}_\ell G$-module.

(ii) Deduce that the consituents of $R_e(\theta)$ and $R_e(\theta')$ are not in the same block unless θ and θ' take the same values on ℓ'-elements.

(d) Generalize these results to the case where ℓ divides $q+1$ using $\mathbf{T}^{sF}$ instead of T and $R\Gamma_c(\widetilde{\mathbf{X}}(s), \overline{\mathbb{F}}_\ell)$ instead of $\overline{\mathbb{F}}_\ell[G/U]$.

WARTHOG 2018, Lecture V-2

Main Exercise 2. Let A be a finite dimensional k-algebra such that $\operatorname{Irr} A = \{k, S\}$. We assume that the projective and injective indecomposable modules have the following shape:

$$P_k = I_k = \begin{matrix} k \\ S \\ k \end{matrix} \quad \text{and} \quad P_S = I_S = \begin{matrix} S \\ k \\ S \\ k \\ \vdots \\ S \end{matrix}$$

(a) Compute $\operatorname{Hom}_A(P, Q)$ for all projective indecomposable modules P and Q. Draw the quiver of A with relations.

(b) Let $f : P_S \longrightarrow P_k$ be a non-trivial morphism of A-modules. We form the 2-term complex

$$C = \cdots 0 \longrightarrow P_S \oplus P_S \xrightarrow{(f,0)} P_k \longrightarrow 0 \cdots$$

(i) Show that $\operatorname{Hom}_A(C, C[n]) = 0$ if $|n| > 1$.

(ii) Show that any morphism of complexes $C \to C[1]$ or $C[1] \to C$ is null-homotopic.

(iii) Deduce that C is a tilting complex for A and that A is derived equivalent to $\operatorname{End}_{D^b(A)}(C)$.

(c) Determine the structure of the projective indecomposable modules of the algebra $B = \operatorname{End}_{D^b(A)}(C)$.