Main Exercise 1. We assume $G = \text{SL}_2$ and $\ell > 2$ ($\ell \neq p$).

Assume first that $\ell$ divides $q - 1$.

(a) Determine the irreducible characters with trivial defect.

(b) Let $\theta, \theta' \in \text{Irr} T$ such that $\theta$ and $\theta'$ take the same values on $\ell'$-elements. Show that the irreducible constituents of $R_e(\theta)$ and $R_e(\theta')$ are in the same block.

(c) Given $\theta \in \text{Irr} T$, let

$$e_\theta = \frac{1}{|T'|} \sum_{t \in T'} \theta(t) t^{-1}.$$ 

(i) Show that $\overline{F}_\ell[G/U]e_\theta$ is a projective $\overline{F}_\ell G$-module.

(ii) Deduce that the constituents of $R_e(\theta)$ and $R_e(\theta')$ are not in the same block unless $\theta$ and $\theta'$ take the same values on $\ell'$-elements.

(d) Generalize these results to the case where $\ell$ divides $q + 1$ using $T^{s\overline{F}}$ instead of $T$ and $R\Gamma_e(\overline{X}(s), \overline{F}_\ell)$ instead of $\overline{F}_\ell[G/U]$. 
Main Exercise 2. Let $A$ be a finite dimensional $k$-algebra such that $\text{Irr} A = \{k, S\}$. We assume that the projective and injective indecomposable modules have the following shape:

$$
\begin{array}{c}
P_k = I_k = S \\
\end{array}
\quad and 
\begin{array}{c}
P_S = I_S = S \\
\end{array}
$$

(a) Compute $\text{Hom}_A(P, Q)$ for all projective indecomposable modules $P$ and $Q$. Draw the quiver of $A$ with relations.

(b) Let $f : P_S \rightarrow P_k$ be a non-trivial morphism of $A$-modules. We form the 2-term complex

$$
C = \cdots 0 \rightarrow P_S \oplus P_S \xrightarrow{(f,0)} P_k \rightarrow 0 \cdots
$$

(i) Show that $\text{Hom}_A(C, C[n]) = 0$ if $|n| > 1$.

(ii) Show that any morphism of complexes $C \rightarrow C[1]$ or $C[1] \rightarrow C$ is null-homotopic.

(iii) Deduce that $C$ is a tilting complex for $A$ and that $A$ is derived equivalent to $\text{End}_{D^b(A)}(C)$.

(c) Determine the structure of the projective indecomposable modules of the algebra $B = \text{End}_{D^b(A)}(C)$. 
