Exercise 5.1. Let $I = (xz, yz) \subset k[x, y, z]$. Determine the minimal prime ideals containing $I$ and compute their codimensions.

Exercise 5.2. Let $I$ be a finitely generated ideal in an infinite polynomial ring $R$. Show that $I$ has infinite dimension. [This is intuitively obvious, but to actually prove it you have to exhibit long chains of prime ideals containing $I$.]

Exercise 5.3. Find three non-zero polynomials $f_1, f_2, f_3$ in $k[x, y, z]$ that are pairwise coprime but that do not form a regular sequence.
Lecture 5: additional exercises

Exercise 5.4. Let $p$ be a prime of finite codimension $c$ in a polynomial ring (over a field). Show that $p$ is finitely generated. [Hint: to do the $c = 1$ case, contract down to a finite variable polynomial ring containing a non-zero element of $p$ and then extend back. The general case can be proved by induction on $c$, using a similar idea.]

Exercise 5.5. Prove the key proposition from lecture: extending ideals from a polynomial ring to a large polynomial ring does not change codimension. First treat the case where $I$ is prime. Then do the general case; you may need to make use of Exercise 5.4 in the general case.