1. Fill in the details in the computation of \( C(\sigma_1^n) \) that we did in the lecture. (In particular, check that the morphisms are correct.) Explain how this complex is related to the complex of problem 4 on the preliminary exercises. Use it to compute \( \text{HHH}(T(2, n)) \).

2. This exercise is about computing Hom spaces of Bott-Samelsons.
   
   (a) Show that the elementary Bott-Samelson bimodule \( B_i \) is self-dual (as an \( R-R \) bimodule). Deduce that if \( B \) and \( B' \) are Bott-Samelsons, then
   
   \[
   \text{Hom}(B, B') = \text{HHH}^0(B_r \otimes B'),
   \]
   
   where \( B_r \) is the reverse of \( B \); if \( B = B_{i_1} \cdots B_{i_k} \), then \( B_r = B_{i_k} \cdots B_{i_1} \). Explain how this is related to problem 2 of the preliminary exercises.
   
   (b) Use the MOY rules to compute the graded dimension of \( \text{Hom}(B_s, B_t) \), where \( s, t \in S^3 \).
   
   (c) Describe the space \( \text{Hom}(B_{12}, B_{21}) \) explicitly.

3. Consider the Rouquier complex of a positive crossing \( C(\sigma) \). Show that the maps \( X_1, X'_2 : C(\sigma) \to C(\sigma) \) given by multiplication by \( X_1 \) and \( X'_2 \) are homotopic. Deduce that \( \text{HHH}(L) \) can be naturally viewed as a module over \( \mathbb{Z}[X_1, \ldots, X_l] \), where \( l \) is the number of components of \( L \).

4. This exercise is about the full twist on \( n = 3 \) strands.
   
   (a) Let \( H = \sigma_1 \sigma_2 \sigma_1 \) be the half-twist. Compute \( C(H) \). (Hint: after cancellation, all relevant morphism spaces are 1-dimensional.)
   
   (b) By comparing with \( C(\sigma_2 \sigma_1 \sigma_2) \), deduce that \( \text{HHH} \) is invariant under the third Reidemeister move.
   
   (c) Let \( T = H^2 \) be the full twist. Compute the minimal complex for \( C(T) \) at the level of objects.
   
   (d) Consider the minimal form of \( C(T^k) \), with the homological grading renormalized so that the lowest nonzero term in the complex is in homological grading \( 0 \). Show by induction that the only objects appearing in homological degrees \( 0, \ldots, k \) are copies of \( B_{121} \).

5. Let \( \overline{\mathfrak{H}}_c \) and \( \mathfrak{H}_c \) be the rational Cherednik algebras associated to the Lie algebras \( \mathfrak{gl}_n \) and \( \mathfrak{sl}_n \) with parameter \( c \). Show that \( \overline{\mathfrak{H}}_c \simeq D \otimes \mathfrak{H}_c \), where \( D \) is an algebra generated by two elements \( x = \frac{1}{n} \sum x_i \) and \( y = \frac{1}{n} \sum y_i \) satisfying \( [x, y] = 1 \). Similarly, show that \( \overline{M}_c \simeq M_c \otimes \mathbb{C}[x] \). Discuss how this is compatible with the relation between reduced and unreduced versions of \( \text{HHH} \).
6. Let $\mathfrak{P}_c$ and $\mathfrak{H}_c$ be the rational Cherednik algebras associated to the Lie algebras $\mathfrak{gl}_n$ and $\mathfrak{sl}_n$ with parameter $c$. Find explicit polynomials of degree $m$ in the polynomial representation of $\mathfrak{P}_{m/n}$ which are annihilated by the action of the Dunkl operators when $m/n = 1/n$ and $m/n = 2/3$. (As discussed in the lecture, these generate the ideal $I_{m/n}$.)

7. The goal of this exercise is to give a hands-on proof of the symmetry of finite dimensional representations of $\mathfrak{H}_c$.

   (a) Let $\mathbf{h} = \frac{1}{2} \sum x_i y_i + y_i x_i, \mathbf{x} = \frac{1}{2} \sum x_i^2$, and $\mathbf{y} = \frac{1}{2} \sum y_i^2$. Show that $\mathbf{h}, \mathbf{x}$ and $\mathbf{y}$ generate an action of $\mathfrak{sl}_2$ on $\mathfrak{H}_c$. If $p$ is a homogenous element of $\mathfrak{P}_c$, show that $[\mathbf{h}, p] = (\deg p)p$.

   (b) Use the decomposition from exercise 5 to construct analogous elements $\mathbf{h}, \mathbf{x}$ and $\mathbf{y}$ generating an action of $\mathfrak{sl}_2$ on $\mathfrak{H}_c$.

   (c) If $v$ is a homogenous element of $M_{m/n}$, show that $\mathbf{h} \cdot v = (\deg v)v$, where $\deg v$ is the degree of $v$ as a polynomial minus $(m-1)(n-1)$. (Hint: first do it for $v = 1$.)

   (d) Deduce that if $M_{m/n}$ is finite dimensional, it admits an involution $\iota$ which commutes with the action of $S_n$ and satisfies $\deg \iota(v) = -\deg v$.

**Supplementary Exercises**

8. Using the formula for $[D_j, X_i]$ we derived in class, prove that $[[D_i, D_j], X_k] = 0$. Deduce that $[D_i, D_j] = 0$.

9. Let $\mathbf{B}$ be a Bott-Samelson diagram on $n$ strands. Show that $HH(\mathbf{B})$ is supported in $a$-gradings $-n+1, -n+3, \ldots, n-3, n-1$. Deduce the Morton-Franks-Williams inequality: if $\sigma \in \text{Br}_n$, then $HHH(\sigma)$ is supported in $a$ gradings between $w - n + 1$ and $w + n - 1$, where $w$ is the writhe of the braid (the number of positive crossings minus the number of negative crossings.)