1. Describe $\text{Hilb}^n(X)$ explicitly when $X$ is the germ of the singularity given by the equation $x^2 = y^5$. Find the generating series of Poincare polynomials and see that it agrees with the bottom row of $\text{HHH}(T(2, 5))$. Can you do the same thing for $T(3, 4)$?

2. Let

$$X(n, m) \subset \text{Hilb}^n(X) \times \text{Hilb}^{n+m}(X) = \{(I, J) \mid M \cdot I \subset J \subset I\},$$

and let $\pi : X(n, m) \to \text{Hilb}^n(X)$ be the natural projection. Show that if $I \in \text{Hilb}_r^n(X)$ (so its minimal number of generators is $r$), then $\pi^{-1}(I) = \text{Gr}(m, r)$ is the Grassmanian of $m$-planes in $\mathbb{C}^r$. Deduce that the generating series

$$\sum_{n, r} q^{2n} \prod_{i=1}^r (1 + t^{2i-1}a^2) \mathcal{P}(\text{Hilb}_r^n(X)) = \sum_{n, m} q^{2n} t^{m^2} a^{2m} \mathcal{P}(X(n, m))$$

3. Let $X$ be the germ of the singularity given by the equation $x^n = y^m$. Show that any ideal in $\mathcal{O}_X$ is generated by at most $\min(n, m)$ generators.

4. Let $S_{n, m} \subset R$ be the semigroup generated by $n$ and $m$. Show that

$$\sum_{s \in S_{n, m}} q^{2s} = q^{\mu-1} \mathcal{P}(T(n, m))|_{a=1}.$$