WARTHOG 2016
Exercises for Tuesday morning

1. Let \( \mathfrak{h} \) be the Cartan subalgebra of \( \mathfrak{gl}_n \), viewed as a representation of the Weyl group \( S_n \). Let \( \mathcal{H}_{m/n} = \text{Hom}_S(\Lambda^\bullet \mathfrak{h}, \mathcal{H}_{m/n}) \). Show that \( \mathcal{H}_{m/n} \cong \mathcal{H}_{m/n} \otimes \mathbb{C}[x] \otimes H^*(S^1) \).

2. Compute the homology of \( d_2 : \mathcal{H}_{\infty,2} \to \mathcal{H}_{\infty,2} \); check that it agrees with \( Kh(T(2, \infty)) \).

3. Compute the homology of \( d_2 : \mathcal{H}_{\infty,3} \to \mathcal{H}_{\infty,3} \), which should correspond to \( Kh(T(3, \infty)) \).

4. Let \( R = \mathbb{C}[x_1, \ldots, x_n] \), and let \( R_{S_n} = R/R_{S_n}^> \) be the ring of coinvariants. By considering partial derivatives of symmetric functions, find explicit polynomials generating \( \text{Hom}_{S_n}(\mathfrak{h}^*, R_{S_n}) \).

5. Suppose \((m, 3) = 1\), and consider the BGG resolution of \( L_{m/3} \). Describe how each term in the resolution decomposes as a graded representation of \( S_3 \). Use your answer to compute the graded dimension of \( L_{m/3} \).

6. Let \( \lambda \) be a partition of \( n \), and let \( \lambda_1, \ldots, \lambda_r \) be the partitions of \( n+1 \) obtained by adding a single box to the Young diagram of \( \lambda \). Let \( c_\lambda \) be the eigenvalue of the full twist on the idempotent \( e_\lambda \). Show that \( c_{\lambda_1}, \ldots, c_{\lambda_r} \) are all distinct.

7. Compute the action of \( T_i \in H_n \) on \( V^{\lambda}_{(n)} \) of \( H_n \). Explain what Kazhdan and Lusztig’s theorem relating representations of \( H_n \) and \( S_n \) means in this case.

8. Find the central idempotents \( e_{(3)}, e_{(2,1)}, e_{(1,1,1)} \in H_3 \). Compute their products with the Kazhdan-Lusztig basis elements \( C_s \). What is the action of \( T_i \) on \( e_{2,1} H_3 \)?

9. Show that the number of crossingless planar \( n \)-tangles in with \( k \) turnbacks is equal to the dimension of \( V_\lambda \), where \( \lambda \) is the partition of height 2 and length \( n-k \).

10. Find Bott-Samelson diagrams corresponding to the crossingless planar tangles below.

11. Look up Mark Goresky’s tables of Kazhdan-Lusztig polynomials of Schubert varieties. Work out the map \( \pi : S_4 \to \{\text{partitions of 4}\} \) with their help.