WARTHOG 2016
Exercises for Tuesday morning

1. Let $\overline{\mathfrak{h}}$ be the Cartan subalgebra of $\mathfrak{gl}_n$, viewed as a representation of the Weyl group S_n). Let $\overline{\mathcal{H}}_{m/n} = \mathrm{Hom}_{S_n}(\Lambda^* \overline{\mathfrak{h}}, \overline{\mathbf{H}}_{m/n})$. Show that $\overline{\mathcal{H}}_{m/n} \simeq \mathcal{H}_{m/n} \otimes \mathbb{C}[x] \otimes H^*(S^1)$.

2. Compute the homology of $d_2 : \overline{\mathcal{H}}_{\infty,2} \to \overline{\mathcal{H}}_{\infty,2}$; check that it agrees with $Kh(T(2,\infty))$. Compute the homology of $d_2 : \mathcal{H}_{\infty,3} \to \mathcal{H}_{\infty,3}$, which should correspond to $Kh(T(3,\infty))$.

3. Let $R = \mathbb{C}[x_1, \dots, x_n]$, and let $R_{S_n} = R/R^{S_n}_{>0}$ be the ring of coinvariants. By considering partial derivatives of symmetric functions, find explicit polynomials generating $\mathrm{Hom}_{S_n}(\mathfrak{h}^*, R_{S_n})$.

4. Suppose $(m,3) = 1$, and consider the BGG resolution of $L_{m/3}$. Describe how each term in the resolution decomposes as a graded representation of S_3. Use your answer to compute the graded dimension of $L_{m/3}$.

5. Let λ be a partition of n, and let $\lambda_1, \dots, \lambda_r$ be the partitions of $n+1$ obtained by adding a single box to the Young diagram of λ. Let c_λ be the eigenvalue of the full twist on the idempotent $\mathbf{e}_\lambda$. Show that $c_{\lambda_1}, \dots, c_{\lambda_r}$ are all distinct.

6. Compute the action of $T_i \in H_n$ on $V^q_{(n)}$ of H_n. Explain what Kazhdan and Luzstig's theorem relating representations of H_n and S_n means in this case.

7. Find the central idempotents $\mathbf{e}_{(3)}, \mathbf{e}_{(2,1)}, \mathbf{e}_{(1,1,1)} \in H_3$. Compute their products with the Kazhdan-Lusztig basis elements C_s. What is the action of T_i on $\mathbf{e}_{2,1}H_3$?

8. Show that the number of crossingless planar n-tangles in with k turnbacks is equal to the dimension of V_λ, where λ is the partition of height 2 and length $n-k$.

9. Find Bott-Samelson diagrams corresponding to the crossingless planar tangles below.

10. Look up Mark Goresky's tables of Kazhdan-Lusztig polynomials of Schubert varieties. Work out the map $\pi : S_4 \to \{\text{partitions of } 4\}$ with their help.