MONDAY EXERCISE 1

Consider the polynomial
$$f = 6x^2 + 4xy - 3y^2 + 11x - y + 12 \in \mathbb{Q}[x^{\pm 1}, y^{\pm 1}],$$
where $\mathbb{Q}$ has the 3-adic valuation.

(1) Compute $\operatorname{trop}(f)$.

(2) Draw a picture of $V(\operatorname{trop}(f)) \subset \mathbb{R}^2$.

(3) Give two other polynomials $g, h \in \mathbb{Q}[x^{\pm 1}, y^{\pm 1}]$ with $V(\operatorname{trop}(g)) = V(\operatorname{trop}(h)) = V(\operatorname{trop}(f))$. Can you find one with $\operatorname{trop}(g) \neq \operatorname{trop}(f)$?

(4) Consider the lines given by the equations $h_1 = 0$ and $h_2 = 0$, where
$$h_1 = x - 27$$
and
$$h_2 = x - 1$$
What are $\operatorname{trop}(V(\langle f, h_1 \rangle))$ and $\operatorname{trop}(V(\langle f, h_2 \rangle))$? How do these relate to $V(\operatorname{trop}(f)) \cap V(\operatorname{trop}(h_1))$ and $V(\operatorname{trop}(f)) \cap V(\operatorname{trop}(h_2))$?