THURSDAY EXERCISE 2
BERKOVICH SPACES, PART II

Let $K = \mathbb{C}(\!(t)\!)$ be the field of formal Laurent series equipped with the non-Archimedean norm $|-| : \mathbb{C}(\!(t)\!) \longrightarrow \mathbb{R}_{\geq 0}$ given by $\left|\sum_{n=n_0}^{\infty} a_n t^n\right| = e^{-n_0}$ when $a_{n_0} \neq 0$. Let

$$X \;=\; \operatorname{Spec} \mathbb{C}(\!(t)\!)[w] \;=\; \mathbb{A}^1_{\mathbb{C}(\!(t)\!)}$$

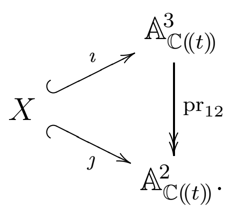
be the affine line over $\mathbb{C}(\!(t)\!)$. Let

$$\imath : \; X = \mathbb{A}^1_{\mathbb{C}(\!(t)\!)} \hookrightarrow \mathbb{A}^3_{\mathbb{C}(\!(t)\!)} = \operatorname{Spec} \mathbb{C}(\!(t)\!)[x, y, z]$$

be the closed embedding dual to the homomorphism taking $x \mapsto w + t$, $y \mapsto w - t$, and $z \mapsto w + 1 - t$. Let

$$\jmath : \; X = \mathbb{A}^1_{\mathbb{C}(\!(t)\!)} \hookrightarrow \mathbb{A}^2_{\mathbb{C}(\!(t)\!)} = \operatorname{Spec} \mathbb{C}(\!(t)\!)[x, y]$$

be the closed embedding obtained by composing $\imath$ with the projection $\mathrm{pr}_{12} : \mathbb{A}^3_{\mathbb{C}(\!(t)\!)} \twoheadrightarrow \mathbb{A}^2_{\mathbb{C}(\!(t)\!)}$ that forgets the z-coordinate. Note that we have a commutative diagram



(1) Draw $\mathrm{Trop}(X, \imath)$ inside $\mathrm{Trop}(\mathbb{A}^3_{\mathbb{C}(\!(t)\!)})$, and draw $\mathrm{Trop}(X, \jmath)$ inside $\mathrm{Trop}(\mathbb{A}^2_{\mathbb{C}(\!(t)\!)})$.

(2) Explain what parts of the infinite tree inside the Berkovich affine line $X^{\mathrm{an}} = \mathbb{A}^{1,\mathrm{an}}_{\mathbb{C}(\!(t)\!)}$ collapse under the tropicalization map

$$\mathrm{trop}_\imath : X^{\mathrm{an}} \longrightarrow \mathrm{Trop}(X, \imath),$$

and likewise under tropicalization map

$$\mathrm{trop}_\jmath : X^{\mathrm{an}} \longrightarrow \mathrm{Trop}(X, \jmath).$$

What information in X^{an} is lost under the map

$$\mathrm{Trop}(\mathrm{pr}_{12}) : \mathrm{Trop}(X, \imath) \longrightarrow \mathrm{Trop}(X, \jmath) \quad ?$$