A very brief review of calculus

Derivatives

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

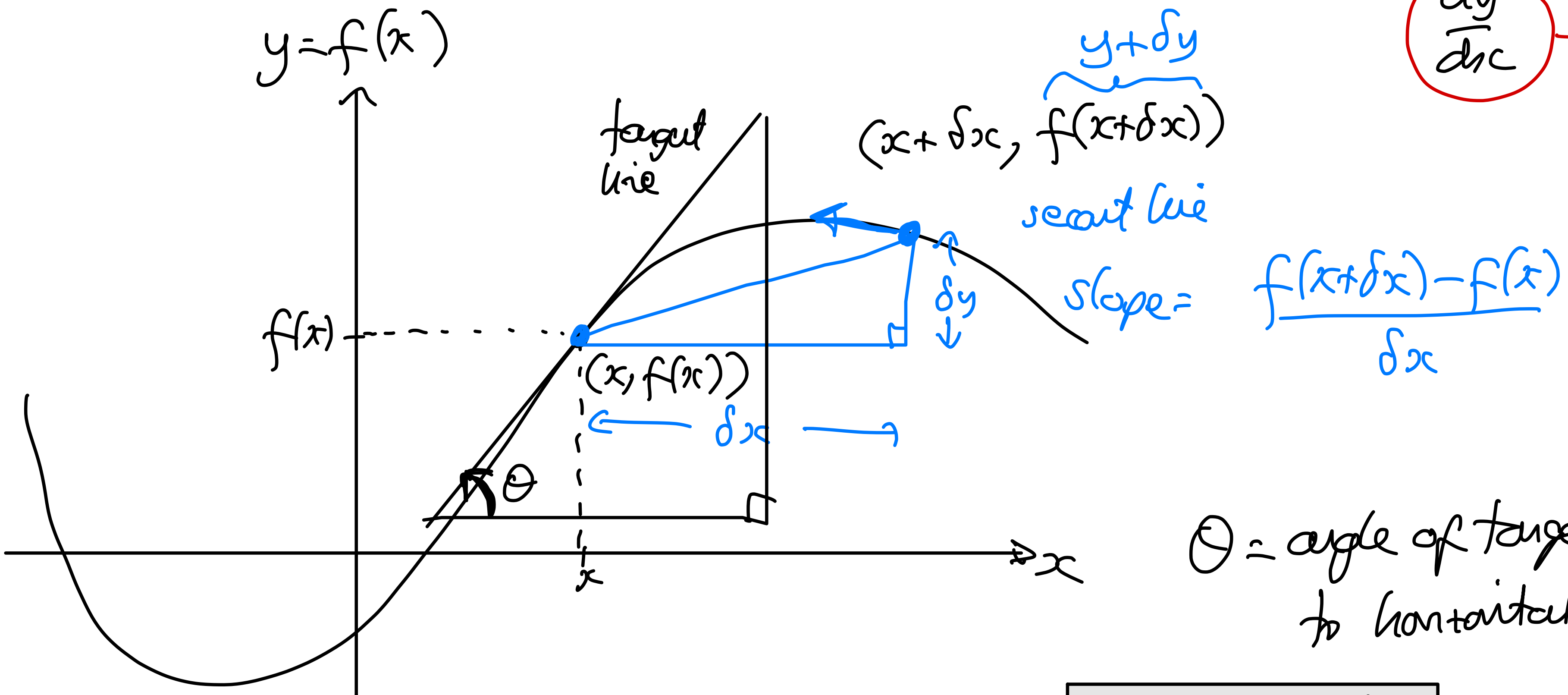
New function whose value at x is slope of tangent line to original graph.

Newton notation

$f'(x)$

Leibniz notation

$\frac{dy}{dx}$



θ = angle of tangent line to horizontal

Slope = $\frac{\text{rise}}{\text{run}}$

$$\boxed{\tan\theta = \frac{dy}{dx}}$$

In first calculus course, you used derivatives to find critical points

Hence maxima & minima.

$f'(x) = 0$

tangent
horizontal

Derivatives of elementary functions are easy to calculate !!!

Basic rule : power rule

any constant power

$$\frac{d}{dx}\left(x^{n}\right) = n x^{n-1}$$

"power down the front, one off the power"

$\frac{d}{dx}$ is instruction to differentiate wrt x

Proof $\frac{d}{dx}\left(x^{\cancel{n}\,3}\right) = \lim_{h \to 0} \frac{(x+h)^{\cancel{n}\,3} - x^{\cancel{n}\,3}}{h}$

When n is a natural number!!!

Just do it when $n=3$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \left(3x^2 + 3xh + h^2\right)$$

$$= 3x^2$$

Binomial Theorem...

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ \vdots \end{array}$$

$$\frac{d}{dx}\left(x^{\pi}\right) = \pi x^{\pi - 1} \ ??$$

what does this mean?

product rule

$(f \cdot g)(x) = f(x)\,g(x)$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

first second

(a.k.a. Leibniz rule)

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

quotient rule

top

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{(g)^2}$$

bottom

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

chain rule

$$(f \circ g)' = (f' \circ g) \cdot g'$$

outside inside

$(f \circ g)(x) = f(g(x))$

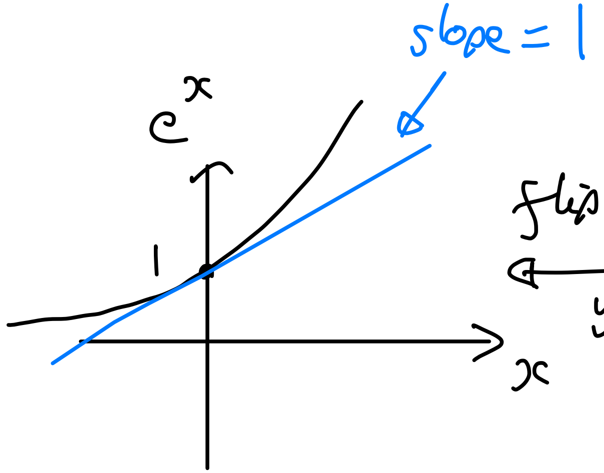
"f after g"

$$\frac{d}{dx}\Big[\overbrace{f(\underbrace{g(x)}_{v})}^{u}\Big] = f'(g(x))\,g'(x)$$

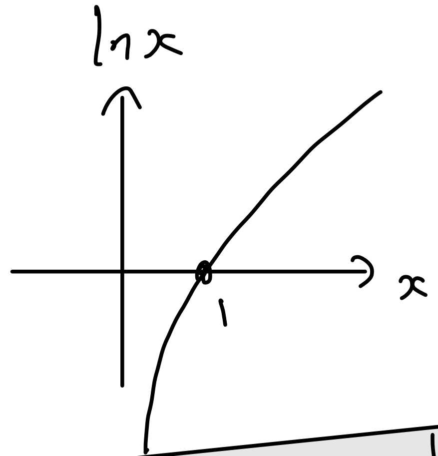
$$\boxed{\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}}$$

<u>Exp & log</u>

$$e^x \xleftrightarrow[\text{functions}]{\text{Inverse}} \ln x \qquad \begin{cases} \ln(e^x) = x \\ e^{\ln x} = x \end{cases}$$



flip in $\longleftrightarrow$ $y = x$



$e = 2.718\ldots$

$$\boxed{\frac{d}{dx}(e^x) = e^x} \qquad \boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

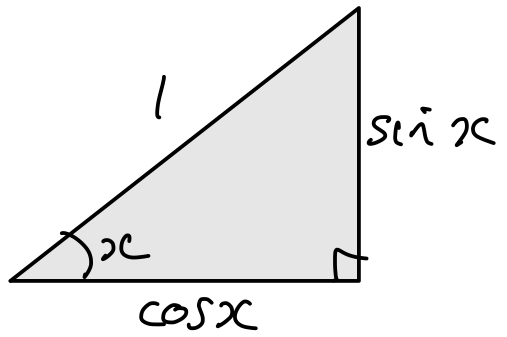
- $\frac{d}{dx}(10^x) = \frac{d}{dx}\left[\left(e^{\ln 10}\right)^x\right] = \frac{d}{dx}\left[e^{(\ln 10)\cdot x}\right] = e^{(\ln 10)\cdot x} \times \ln 10 = (\ln 10) 10^x$

- x^π ? $\quad x^\pi$ means $\left(e^{\ln x}\right)^\pi = e^{\pi \cdot \ln x}$

$$\frac{d}{dx}(x^\pi) = \frac{d}{dx}\left(e^{\pi \ln x}\right) = e^{\pi \ln x} \times \frac{\pi}{x} = \frac{\pi}{x} \cdot x^\pi = \pi x^{\pi - 1} \checkmark$$

Trig. functions

SOHCAHTOA



$$\sin x, \quad \cos x$$

$$\frac{\sin x}{\cos x} = \tan x, \quad \sec x = \frac{1}{\cos x}$$

$$\frac{\cos x}{\sin x} = \frac{1}{\tan x} = \cot x, \quad \operatorname{cosec} x = \frac{1}{\sin x}$$

Pythagoras $\Rightarrow$

Basic trig. identities
$$\begin{cases} \cos^2 x + \sin^2 x = 1 \\ 1 + \tan^2 x = \sec^2 x \\ \cot^2 x + 1 = \operatorname{cosec}^2 x \end{cases}$$

$$\sin^2 x = (\sin x)^2 = \sin x \cdot \sin x$$

$$\sin^{-1} x = (\sin x)^{-1} = \frac{1}{\sin x}$$

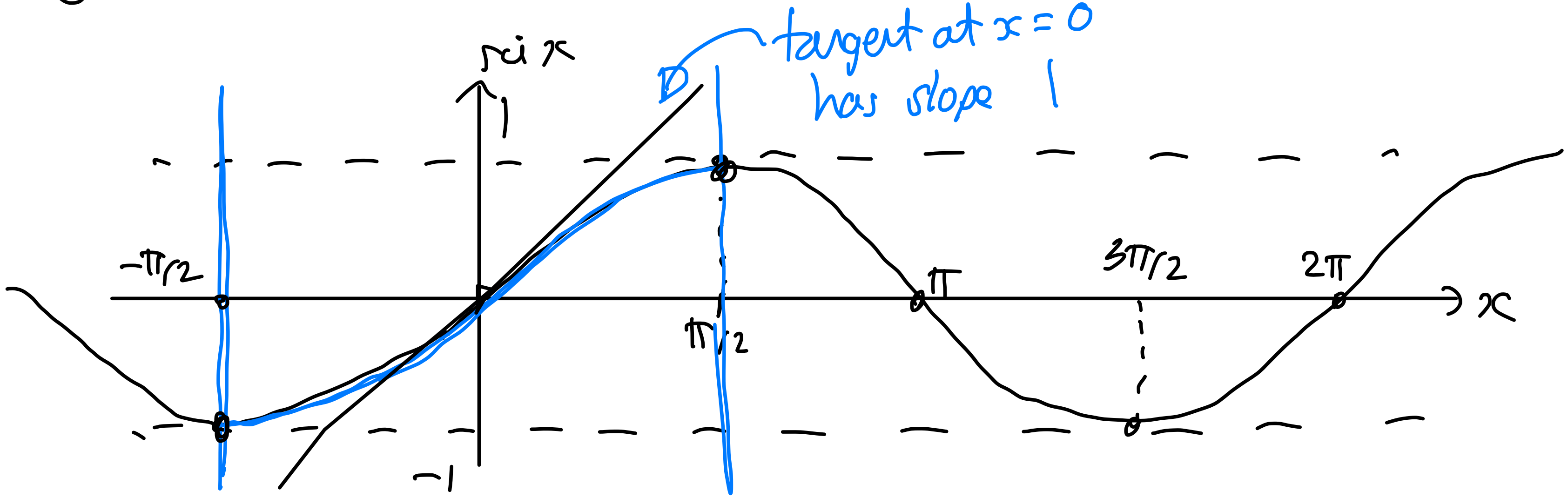
NOT the inverse trig function — I use $\arcsin x$ for that

$\arccos x$

$\arctan x$

$y = \arcsin x \iff \sin y = x$

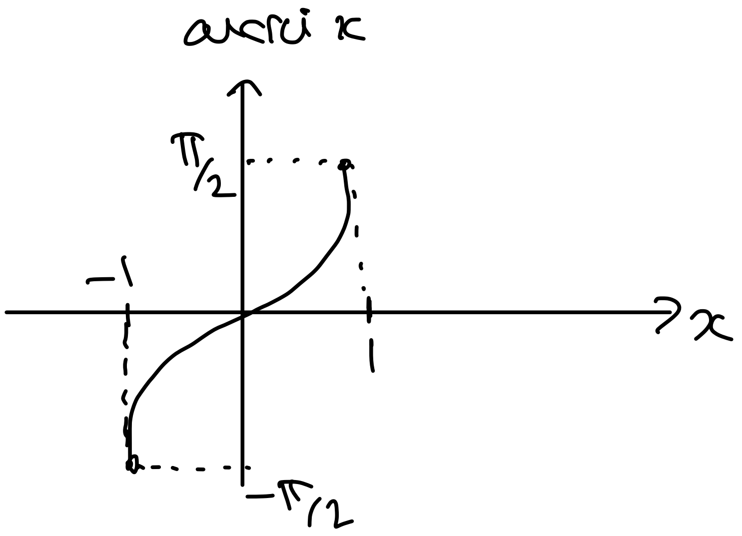
USE RADIANS ALWAYS!!

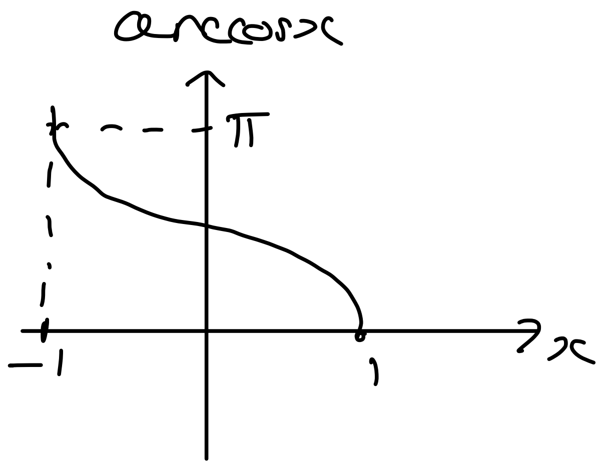


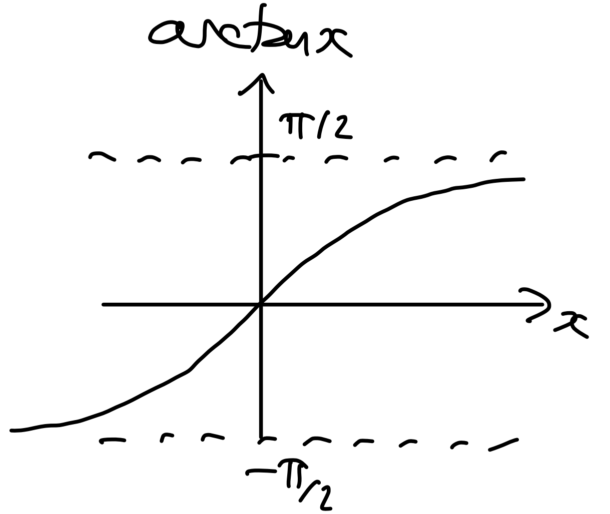
Periodic, period 2π

Not 1-1, so not invertible!!! Need to restrict domain of sin to $[-\pi/2, \pi/2]$

arcsin is inverse function on that domain.







(1) $\boxed{\dfrac{d}{dx}(\sin x) = \cos x}$ $\quad$ $\boxed{\dfrac{d}{dx}(\cos x) = -\sin x}$

(2) $\boxed{\dfrac{d}{dx}(\tan x) = \sec^2 x}$ $\quad$ $\boxed{\dfrac{d}{dx}(\sec x) = \sec x \cdot \tan x}$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x + \sin x \cdot \sin x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

(3) $\boxed{\dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}}$ $\quad$ $\boxed{\dfrac{d}{dx}(\arctan x) = \dfrac{1}{1+x^2}}$

$y = \arcsin x$ $\qquad \dfrac{dy}{dx}$?

$\Updownarrow$

$\sin y = x$

$$\therefore \frac{d}{dx}(\sin y) = \frac{d}{dx}(x) \qquad \therefore \frac{dy}{dx} \cdot \frac{d}{dy}(\sin y) = 1 \qquad \therefore \frac{dy}{dx} \cos y = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$