Some of Newton's physics

$F = ma$

Newton kg m/sec^2

Work = Force × distance

energy

Joules = N·m = kg·m^2/sec^2

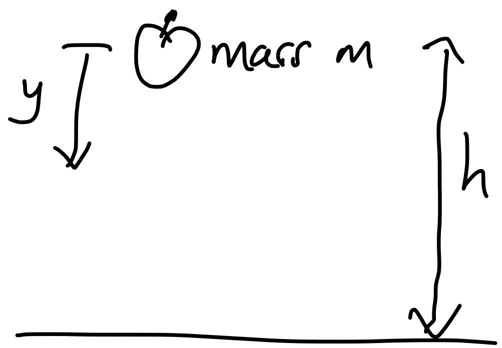
To raise apple from ground to height h takes $\boxed{mgh}$ Joules of energy

P.E.
potential energy

Conservation of energy

All mgh Joules of P.E. becomes K.E. when it hits ground.

$\boxed{\frac{1}{2}mv^2}$ K.E. mass m velocity v



Gravity exerts force mg

$$\Longrightarrow \ddot{y} = g$$

$$\dot{y} = gt + \cancel{c} \qquad \dot{y}(0) = 0 \quad \therefore c = 0$$

$$y = \frac{1}{2}gt^2 + \cancel{d} \qquad y(0) = 0 \quad \therefore d = 0$$

When does the apple hit the ground?

$$\frac{1}{2}gt^2 = h \quad \therefore \quad t = \sqrt{\frac{2h}{g}}$$

What is velocity of apple when that happens?

$$\dot{y} = gt = g\sqrt{\frac{2h}{g}} = \sqrt{2gh} \text{ when it hits.}$$

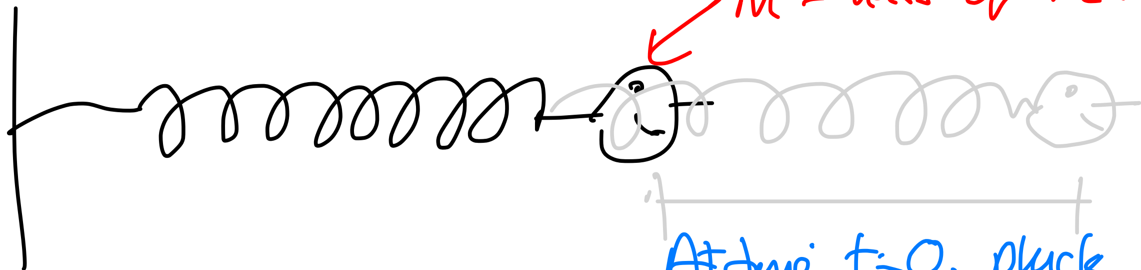
Mass m moving at velocity $v = \sqrt{2gh}$

K.E.? $mgh = \frac{1}{2}mv^2$

$\cancel{\frac{1}{2}} m \cdot \cancel{2} gh$ ✓

Another example

m = mass of head of springy



$x = x(t)$ extension of spring at time t

At time $t=0$, pluck springy to extension x_0 then let go...

Work out equation of motion.

Need Hooke's Law ... $F \propto x$
Force   Extension

$$F = -kx \quad , k \text{ spring constant}$$

Together with $F = ma$, get differential equation

$$m\ddot{x} = -kx \quad \ldots \quad \boxed{\ddot{x} + \frac{k}{m}x = 0}$$

2nd order diff. eq.

$$\therefore \; x(t) = a\cos\left(\sqrt{\tfrac{k}{m}}\,t\right) + b\sin\left(\sqrt{\tfrac{k}{m}}\,t\right) \quad \text{some } a, b \quad \text{(see theorem below)}$$

Set $t=0$, get $a = x_0$ and $\dot{x}(0) = 0$ so $\sqrt{\frac{k}{m}}\,b = 0 \quad \therefore b = 0$

So
$$\boxed{x(t) = x_0\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)}$$

same as K.E. when goes through equilibrium point. Time $t = \frac{\pi}{2}\cdot\sqrt{\frac{m}{k}}$

How much energy is stored in spring at time $t=0$? $\quad \dot{x}(t) = -x_0\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}\,t\right)$

Theorem The general solution of $f''(x)+f(x)=0$ is
$f(x)=a\cos x+b\sin x$ where $a=f(0)$, $b=f'(0)$.

Proof. Note $f(x)=\cos x$ and $f(x)=\sin x$ are solutions of $f''(x)+f(x)=0$
It's a linear homogeneous diff. eq., any linear combination of solutions is also a solution.

$a\cos x+b\sin x$

$g'(x)=f'(x)+a\sin x-b\cos x$

Now let $a=f(0)$ and $b=f'(0)$ and consider $g(x)=f(x)-a\cos x-b\sin x$.
This is a solution of the diff. eq., and $g(0)=0$, $g'(0)=0$

Claim $g(x)=0$ for all x ... hence, $f(x)=a\cos x+b\sin x$, as req'd.

To see this, conceive $[g'(x)]^2+[g(x)]^2=:h(x)$

Then $h'(x)=2g'(x)g''(x)+2g'(x)g(x)=2g'(x)[g''(x)+g(x)]=0$
$\therefore$ $h(x)=c$, constant
But $h(0)=c=0$. Shows $h(x)=0$ for all x ...so $g(x)=g'(x)=0$ //

We've just shown velocity is $-x_0\sqrt{\frac{k}{m}}$ when goes through equilibrium point

$$\therefore K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(x_0\sqrt{\frac{k}{m}}\right)^2 = \frac{1}{2}kx_0^2.$$

$$\boxed{\text{P.E. stored in spring when extension } x = \frac{1}{2}kx^2}$$

Clunky argument ... much better would be to use Work = Force × Distance

$$\text{"} \sum_{x=0}^{x_0} kx \cdot \delta x \text{"}$$

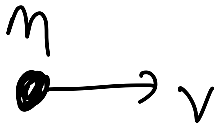
$$\therefore P.E. = \int_0^{x_0} kx \cdot dx$$

$$= \left[\frac{1}{2}kx^2\right]_0^{x_0} = \frac{1}{2}kx_0^2 \quad \checkmark$$

Back to $F=ma$ Newton's Law.

Conservation of energy

Conservation of momentum



momentum $= mv$

Newton's Law really says:

$$\boxed{F = \frac{d}{dt}(mv)}$$

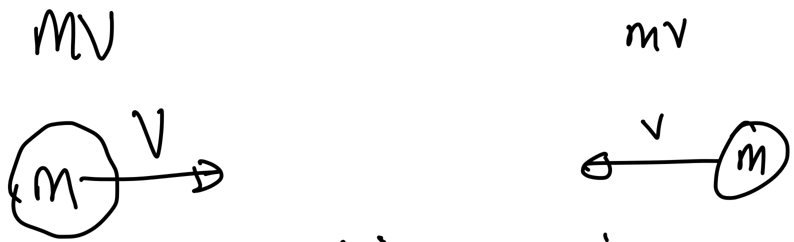
"force is rate of change of momentum"

If m is constant $\frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$

From that you get the idea that it takes force to change momentum.

In a closed system (no external forces) the total momentum is constant over time.

Conservation of momentum.



colliding particles