Math 467, Winter 2015, Prof. Sinclair

Midterm

February 13, 2015

Instructions:

1. Read all questions carefully. If you are confused ask me!
2. You should have 4 pages including this page. Make sure you have the right number of pages.
3. Make sure your name is clearly printed on the first page and your initials are on all subsequent pages.
4. If necessary you may use the back of pages.
5. Box your answers when appropriate.

Name:____________________________________

UO ID:___________________________________

Question:	1	2	3	Total
Points:	10	10	10	30
Score:				

[10 pts] 1. Suppose (X_n) is a doubly stochastic Markov chain with states $\{1, 2, \ldots, N\}$.

(a) Prove that $\pi = (1/N, 1/N, \ldots, 1/N)$ is a stationary distribution.

(b) Give a non-trivial (i.e. non-deterministic) example (of a doubly stochastic Markov chain) for which this is not the only stationary distribution. Demonstrate another stationary distribution for your example, or explain why it has one.

[10 pts] 2. Let $\xi_1, \xi_2, \ldots$ be a sequence of Bernoulli random variables all with $P\{\xi_i = 1\} = p > 0$ and $P\{\xi_i = 0\} = 1 - p > 0$. Let $S_n = S_0 + \xi_1 + \xi_2 + \cdots + \xi_n$. Set $Y_0 = S_0$ and $Y_n = S_n - c_n$ for $n \geq 1$.

(a) For what values of c_n is Y_n a martingale? Justify your answer.

(b) Suppose $S_0 = 10$. Find the probability that $S_n = 5 + pn$ before $S_n = 20 + pn$.

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[10 pts] 3. Suppose the transition matrix for a five state Markov chain (with states $1, 2, 3, 4, 5$) is given by

$$p = \begin{bmatrix} .3 & .4 & 0 & 0 & .3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & .6 & .4 \\ 0 & 0 & 0 & .4 & .6 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(a) Classify each state as either recurrent or transient. Write the state space in the form $T \cup R_1 \cup \cdots \cup R_n$ where T is the set of transient states and each of the R_i are closed irreducible sets of states.

(b) Compute the limiting transition matrix: $\lim p^n$.

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