17. **R as a metric space**

Consider the reals \( \mathbb{R} \) with \( \rho : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \) defined by \( \rho(x, y) = |x - y| \). Show that this definition makes \( \mathbb{R} \) a metric space.

(3 points)

18. **Limits of sequences**

a) Show that a sequence in a metric space has at most one limit.

*hint*: Assume there are two limits, and use the triangle inequality to show that they must be the same.

b) Show that every sequence with a limit is a Cauchy sequence.

(3 points)

19. **Banach space**

Prove Proposition 1 from §4.6, i.e., show that the norm on the dual space \( B^* \) of a Banach space \( B \) as defined in §4.6 def. 4 is a norm in the sense of the norm \( |\ldots| \) defined on \( B \) itself in §4.6 def. 1.

(3 points)

20. **Hilbert space**

a) Show that the norm on a Hilbert space defined by §4.7 def. 1 is a norm in the sense of §4.6 def. 1.

*hint*: Use the Cauchy-Schwarz inequality (§4.7 lemma).

b) Show that the mappings \( \ell \) defined in §4.7 def. 4 are linear forms in the sense of §4.3 def. 1(a).

(3 points)