Electromagnetic Radiation

idea:
- for we know disman
- static relations of $E$'s eq with mms (cl) 
- dynamic relations i vegm (cl)

now let's disman
- dynamic relations in the pm of waves

§1 Review of photons and jamps

1.1 Photons and fields

$\text{d}i \neq \text{d}j \Rightarrow \text{The fields } E \text{ and } B \text{ (not in observables) can be obtained from the photons } \phi \text{ and } \vec{A} \text{ (not in lost) via:}$

\[
\begin{align*}
    E(x,t) &= -\frac{1}{c} \partial_x \phi(x,t) - \frac{1}{c} \partial_t \vec{A}(x,t) \\
    B(x,t) &= \nabla \times \vec{A}(x,t)
\end{align*}
\]

remark:
1. The first two Maxwell (i.e., the homogeneous) are fulfilled.
2. $\text{d}i \neq \text{d}j \Rightarrow \phi, \vec{A}$ on the compons of a 4-vector

\[ A^\mu(x) = (\phi(x), \vec{A}(x)) \]

proposition:
The inhomogeneous $\text{d}i$ eqs (i.e., the $\text{d}^\text{d} \neq \text{d}^\text{d}$)

\[ \partial_\mu \partial_\nu A^\mu(x) - \partial^\mu \partial_\mu A^\nu(x) = \frac{\mu_0}{c} \vec{J}(x) \]

proof:
$\text{d}i \neq \text{d}j \Rightarrow \frac{\mu_0}{c} \vec{J} = \partial_\nu \partial_\mu A^\mu(x) = \partial_\nu \partial_\mu A^\nu(x) - \partial_\mu \partial_\nu A^\mu(x)$
whe... \[ \square \Phi = \Phi - \frac{1}{c^2} \partial_t \Phi = \frac{\nu - \Phi}{c^2} \]

when \[ \square = \frac{1}{c^2} \partial_t \partial_t - \nabla^2 \]

**Proof:** \[ \Phi = (c^2 \Phi) \]
\[ \square \Phi = \frac{\nu - \Phi}{c^2} \]
\[ \partial_t \Phi = \frac{\nu - \Phi}{c^2} \]

\[ \Rightarrow \partial_t \partial_t \Phi = \frac{1}{c^2} \partial_t \partial_t \Phi = \square \Phi \]

Remark: (3) In the static case, (8') simplifies to

\[ \nabla^2 \Phi = -\frac{4\pi j}{c} \quad \text{Point's eq.} \]
\[ \nabla^2 \Phi = \frac{4\pi}{c^2} \quad \text{M. eq.} \]

Remark: (4) In vacuum, (8') simplifies to

\[ \square \Phi \Phi + \frac{\nu}{c} \Phi = 0 \]

In a lorentz gauge (\( \partial_j A^j \equiv 0 \), in bulk) further

simplifies to the wave eq.

\[ \square \Phi \Phi = 0 \quad \text{US eq.} \]

**1.2 Gauge invariance**

U. 1.2.4: The potentials are not unique

\[ \partial_j A^j \equiv 0 \quad \text{in a lorentz gauge} \quad \text{("driving a gauge")}. \]
Pseudo deriving on

(1) Lorentz gauge  \[ \partial_\mu A^\mu(x) = 0 \]

or  \[ \frac{1}{c} \partial_\nu \phi(x,t) + \vec{\nabla} \cdot \vec{A}(x,t) = 0 \]

(2) Vector gauge  \[ \vec{\nabla} \cdot \vec{A}(x) = 0 \]

cf. Problem 8

remark: (1) Some books call this 'Lorentz gauge', ed some call it 'vector gauge' .

(2) Another possibility is to choose \( \phi(x) = 0 \). This also sometimes is called 'vector gauge' .

(3) 4 potentials + 1 constant \( \rightarrow \) 3 fields determine

the 6 observable fields \( \vec{E}, \vec{B} \).

proposition: To Lorentz gauge, the eq. of motion for the potentials, § 1.1 (a'), becomes

\[
\begin{align*}
\Box \phi &= \frac{4\pi}{c} J^\mu \\
\Box A^\mu &= \frac{4\pi}{c} \phi^\mu
\end{align*}
\]

(cf.)

proof: Lorentz gauge \( \rightarrow \vec{\nabla} \cdot \vec{A} = -\frac{1}{c^2} \partial_\mu \phi \)

\[ \rightarrow -\frac{1}{c} \partial_\nu \vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \partial_\mu \phi \]

\( \rightarrow \) (2) follows immediately from § 1.1 (a')

work: Once we choose Lorentz gauge, it is maintained under time evolution.

\[ \Box \partial_\mu A^\mu = \left( \frac{4\pi}{c} \right) \partial_\mu J^\mu = 0 \] by using Maxwell equations § 21

Remark: (4) \( \Box J^\mu \rightarrow \partial_\mu J^\mu = 0 \) is not an independent

while, but follows from the field eqs.
proposition 2: When there is a source term in the eq of motion becomes
\[ \Box \Phi = \frac{4\pi}{c^2} \frac{\partial}{\partial t} - \frac{i}{c} \frac{\partial}{\partial x} \Phi \]
\[ \Box^2 \Phi = -\frac{4\pi}{c^2} \Phi \] (18)

proof: \[ \Box \Phi = \frac{4\pi}{c^2} \frac{\partial}{\partial t} - \frac{i}{c} \frac{\partial}{\partial x} \Phi \]

\[ \Box^2 \Phi = \left( \frac{4\pi}{c^2} \frac{\partial}{\partial t} - \frac{i}{c} \frac{\partial}{\partial x} \right)^2 \Phi = \left( \frac{4\pi}{c^2} \frac{\partial}{\partial t} \right)^2 \Phi + \left( \frac{i}{c} \frac{\partial}{\partial x} \right)^2 \Phi \]

\[ = \left( \frac{4\pi}{c^2} \frac{\partial}{\partial t} \right)^2 \Phi = 0 \] by long wavelength

Remark: (1) This eq. for \( \Phi \) is now the term of a electromagnetic

\[ \Box^2 \phi = \frac{4\pi}{c^2} \partial^2 \Phi - \frac{i}{c} \partial_x \Phi \]

proof: \[ \Box \phi = \frac{4\pi}{c^2} \frac{\partial}{\partial t} \Phi - \frac{i}{c} \frac{\partial}{\partial x} \Phi \]

\[ = \left( \frac{4\pi}{c^2} \frac{\partial}{\partial t} \right)^2 \Phi = 0 \] by long wavelength

Remark: (2) While gauge to pick is a matter of choice. Right choice will

\[ \Box^2 \phi = \frac{4\pi}{c^2} \partial^2 \Phi - \frac{i}{c} \partial_x \Phi \]

be more or less unimportant for

2.1 The concept of a Green's function

\[ \text{under an inhomogenous wave eq.} \]

\[ \Box f(x,t) = i(x,t) \]

will \( i(x,t) \) a give inhomogenous

\[ \text{def: } i \text{ Green's function } G(x,t) \text{ for the eq. (1) is a while} \]

\[ \Box G(x,t) = \delta(x) \delta(t) \] (28)

Remark: This is the wave eq. (1) will a special

inhomogenous \( i(x,t) = \delta(x) \delta(t) \)

\[ = \delta(x) \delta(t) \] (28)
Proposition: Let $G(x,t)$ be a solution of (3.1). Then

$$f(x, t) = \int dx' dt' G(x-x', t-t') \delta(x', t')$$

is a solution of (3).

Proof: \[ f(x, t) = (dx'/dt') G(x-x', t-t') \delta(x', t') = \int dx' dt' \delta(x-x', t-t') \delta(x', t') = \delta(x, t) \]

2.2 Green's function for the wave equation

Under a Fourier basis of (2.1) (40) will reduce to $t$:

$$\int dt e^{iwt} G(x, t) = \delta_0(x)$$

$$\Rightarrow \int dt e^{iwt} \delta(t) = \int \delta(t) e^{iwt} \frac{1}{c^2} \frac{\partial^2}{\partial x^2} G(x, t) - \frac{c}{i} \frac{\partial}{\partial t} G(x, t)$$

$$\Rightarrow \int dt \frac{c^2}{1} \frac{\partial^2}{\partial x^2} G(x, t) - \frac{c}{i} \frac{\partial}{\partial t} G(x, t) = \delta(x)$$

Solution by Fourier basis:

$$G_0(x) = \int dx' e^{-i\frac{ct}{c^2}} G(x')$$

$$\Rightarrow \left( \frac{x}{c^2} \right) G_0(x) = \delta(x)$$

$$\Rightarrow G_0(x) = \frac{1}{x^2 + \omega^2/c^2}$$

Now Fourier basis formula: $PH \xi$ 610: $\Rightarrow \int \frac{dx^2}{(\pi)^2} e^{i\frac{x^2}{2}} \frac{\delta}{\delta^2} \frac{1}{x^2 + (i\omega/c)^2} = \frac{1}{\omega} e^{\xi^2/2} e^{-i\omega t/c}$

when $r = 1/x$
\[ G(x, t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} G_{\omega}(x) = \frac{1}{4\pi^2} \int \frac{d\omega}{i\omega} \int \frac{d\omega'}{i\omega'} e^{-i(\omega + \omega')t} = \frac{1}{4\pi^2} \int \frac{d\omega}{i\omega} \delta(t - \frac{\omega}{c}) \]

**Union:** The defining eq. for the Green's fn., \( f_{2.1}(x,t) \), has two solutions:
\[ G_\pm(x,t) = \frac{1}{4\pi^2} \delta \left(t \mp \frac{\omega}{c}\right) \quad \text{for} \quad \omega = 1 \]

**Remark:** (1) Write a time-dependent point source, \( i(x,t) = i(t) \delta(x) \)

\[ f_{\pm}(x_1,t) = \int dx' dt' \frac{1}{4\pi^2|x-x'|} \delta \left(t-t' \mp \frac{\omega}{c}\right) i(t') \delta(x-x') \]
\[ = \frac{1}{4\pi^2} \int dt' \delta \left(t \mp \frac{\omega}{c} - t'\right) i(t') \]
\[ = \frac{1}{4\pi} i \left(t \mp \frac{\omega}{c}\right) \]

**Note:** If the source \( i(t') \) is active at time \( t' \), then the field response occurs at a time \( t = t' \mp \frac{\omega}{c} \) for the solution \( f_\pm \).

**Def.:** \( G_+ \) is called **retarded** Green's fn., \( G_- \) is called **advanced** Green's fn.

**Ex.:** **Causality**
A physical response cannot precede the echo of the source.

**Remark:** Only the retarded value is physical!
work: (2) Advanced 2nd fols on combless new pl in OAT, boit in high-energy physics ed in statistical mech

2.1 The retarded potbils

Derb to the new eq for \( A \) and \( \phi \), \( \S 1.2 \).

\( \S 2.1 \) proposition + \( \S 2.2 \) -->

\[
\phi(x,t) = \int \delta \left( x - x', t - t' \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} dt' dx'
\]

\[\Rightarrow \phi(x,t) = \int \delta \left( x - x', t - t' \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) dt' \]

\[\Rightarrow \phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \]

\( \Phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \]

\( \Phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \)

\[
\Phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \]

\[\Phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \]

\( \Phi(x,t) = \int \delta \left( x - x', t - x^2 \right) \frac{1}{\sqrt{1 - \frac{x'^2}{c^2}}} \delta \left( \frac{x'^2}{c^2} - x^2 \right) \)

Mark: (1) (4), (4*) on called retarded potbils

(2) The time delay \( \delta t = \frac{x^2}{c^2} \) corresponds to the time it takes a signal to travel from point \( x \) to point \( x' \) with velocity \( c \).

(2) (4), (4*) are analogous to Poisson's principle in statistical con, cf \( \S 2.2 \), 2.6. New concept involved by dynamic forces, finite propagation velocity causes retardation.

Problem 35

Problem 36

Problem of a moving along
\[ a = 0.2 \]

**Proof:**

\[ d = \frac{1.5}{1.5 + 1.5} = \frac{1.5}{3} = 0.5 \]

\[ a + b \cdot (c + d) = a + b \cdot 2 = a + 2 \]

\[ a \cdot b = 1 \times 2 = 2 \]

\[ a + b + c + d = 0.5 + 2 + 1 = 3.5 \]

\[ a + b + c - d = 0.5 + 2 + 1 - 1.5 = 1.5 \]

\[ a - b = 0.5 - 2 = -1.5 \]

\[ a + b - c + d = 0.5 + 2 - 1 = 1.5 \]

\[ a - b - c - d = 0.5 - 2 - 1 = -2.5 \]

**Remark:**

\[ \text{Hence:} \quad A = \frac{1}{2} a + b \cdot (c + d) = \frac{1}{2} \times 2 = 1 \]

\[ A + B = a + b + A = a + b + 1 = 3 \]

\[ A \cdot B = a \cdot b + B = a \cdot 2 + B = 2 + B \]

\[ A + B - C = a + b + A - c = a + b + 1 - 1 = 2 \]

\[ A \cdot B - C = a \cdot b - c = a \cdot 2 - c = 2 - c \]

\[ A + B + C = a + b + A + c = a + b + 1 + 1 = 4 \]

\[ A \cdot B + C = a \cdot b + c = a \cdot 2 + c = 2 + c \]
Reduction by him dependent terms

1.1 Asymptotic potentials and fields

Under the extended potentials, § 2.2 (x), (y), et longe distant
\( r = |x| \) per un longe et short distant:

\[
|x - y| = \sqrt{r^2 - 2x \cdot y + y^2} = r \sqrt{1 - 2x \cdot y/r + o(1/r^2)} = r - \frac{x \cdot y}{r} + o(1/r)
\]

\[
\begin{align*}
\varphi(x, t) &= \frac{1}{r} \left[ \int_{|y| < \frac{r}{2}} \delta_0 \left( \frac{y}{r} \right) f(y, t) + o(1/r^2) \right] \\
\delta(x, t) &= \frac{1}{r} \left[ \int_{|y| < \frac{r}{2}} \delta_0 \left( \frac{y}{r} \right) f(y, t) + o(1/r^2) \right]
\end{align*}
\]

When

\[ t_r = t - \frac{x \cdot y}{r} + \frac{1}{2} \frac{x^2 \cdot y}{r^2} \]

Remark: (1) We have only the body contribution for \( r \to \infty \), which is of \( o(1/r) \)

(2) How may zero to keep it the time argument \( /t \) of \( \varphi \) depends on how rapidly the waves on changing. If \( \varphi \)

is the linear problem of the wave, at the wave

drops off for \( t \) a time scale \( \Delta t = c/r \), then

the line \( \frac{x \cdot y}{r} \) in the time argument will be important.

\[
\begin{align*}
\vec{\nabla} \frac{1}{r} f(t_r) &= -\frac{x}{r} \left( \frac{1}{r} \right) \frac{\partial f(t_r)}{\partial t} + o(1/r^2) \\
\end{align*}
\]

Proof:

\[
\begin{align*}
\vec{\nabla} \frac{1}{r} f(t_r) &= \left( \vec{\nabla} \frac{1}{r} \right) f(t_r) + \frac{x}{r} \frac{\partial f(t_r)}{\partial t} \frac{\vec{\nabla} t_r}{r} \\
&= o(1/r^2) + \frac{x}{r} \frac{\partial f(t_r)}{\partial t} \left( \frac{1}{2} \right) \frac{\vec{\nabla} \left| x - \frac{x \cdot y}{r} \right|^2}{r^2} \\
&= -\frac{1}{r} \frac{\partial f(t_r)}{\partial t} \frac{x^2}{r^2} = -\frac{1}{r} \frac{x^2}{r} \frac{\partial f(t_r)}{\partial t} + o(1/r^2)
\end{align*}
\]
Proposition: For free the domains, the fields on join by

\[ \mathbf{E}(\mathbf{x}, t) = -\frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{D}(\mathbf{x}, t) \]

\[ \mathbf{B}(\mathbf{x}, t) = \nabla \times \mathbf{A}(\mathbf{x}, t) \]

Remark: (1) This implies \( \mathbf{E} \parallel \mathbf{B} \), and \( \mathbf{E} \times \mathbf{B} \) from a right-handed orthogonal right.

(4) The fields fall off as \( 1/r \), in the opposite to \( 1/r^2 \) in static variables of \( \mathbf{n} \)'s eye.

(41) Then results on independent of the gauge used, by Noether's.

Proof of proposition: \( \mathbf{E} = \nabla \times \mathbf{B} \)

\[ \mathbf{E} = \frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{D}(\mathbf{x}, t) = \frac{1}{c^2} \frac{\partial}{\partial t} \int \mathbf{D}(\mathbf{x}, t) \]

\[ \mathbf{E} = -\frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{D}(\mathbf{x}, t) \]

\[ \mathbf{E} = -\nabla \phi - \frac{i}{c^2} \frac{\partial}{\partial t} \mathbf{A}(\mathbf{x}, t) \]

\[ \mathbf{E} = \nabla \times \mathbf{B} \]

\[ \mathbf{E} = \frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{D}(\mathbf{x}, t) + \frac{i}{c^2} \mathbf{B}(\mathbf{x}, t) \]

\[ \mathbf{E} = \nabla \times \mathbf{B} \]

\[ \mathbf{E} = \nabla \times \mathbf{B} = \nabla \times \left( \mathbf{A}(\mathbf{x}, t) \right) \]

\[ \mathbf{E} = \nabla \times \mathbf{B} = \nabla \times \left( \mathbf{A}(\mathbf{x}, t) \right) \]

\[ \mathbf{E} = \nabla \times \mathbf{B} = \nabla \times \left( \mathbf{A}(\mathbf{x}, t) \right) \]

\[ \mathbf{E} = \nabla \times \mathbf{B} = \nabla \times \left( \mathbf{A}(\mathbf{x}, t) \right) \]
work: (5) A time-dependent electric field drifiting in time-dependent fields everywhere in space (will propagate at the speed of light in free space). This phenomenon is called **radiation**.

(6) For free the wave, the radiation fields $\vec{E}$ and $\vec{B}$

(i) fall off as $1/r$

(ii) are perpendicular to one another and perpendicular to the radial vector from the source to the observer.

(7) The wave must provide the field energy $\rightarrow$ the power flow of the wave!

1.2 The radiated power

$\Delta t \rightarrow$ the energy flux density of the fields is given by the Poynting vector: $\vec{P}(x,t) = \frac{c}{4\pi} \vec{E}(x,t) \times \vec{B}(x,t)$

**Remark:**

1. $\vec{E} \perp \vec{B} \perp \hat{x} \rightarrow \vec{P} \parallel \hat{x}$

2. $[\vec{P}] = \text{mW/m}^2$ per unit area per unit time

3. $\hat{x} \cdot \vec{P} = \text{power per unit area}$

one unit $dA = r^2 d\Omega$ 

will radiate energy into $d\Omega$

$\rightarrow$ the radiated power per unit area is

$$\frac{d\vec{P}}{d\Omega} = r^2 \hat{x} \cdot \vec{P} = r^2 \frac{c}{4\pi} \hat{x} \cdot (\vec{E} \times \vec{B})$$

$$= -r^2 \frac{c}{4\pi} \hat{x} \cdot ((\hat{x} \times \vec{E}) \times \vec{B}) = r^2 \frac{c}{4\pi} \frac{d^2}{d\Omega}$$
§2.1 \[ \text{We need } \delta(x,t) \text{ at the retarded time} \]

rigid reminder: at point \( x \) and time \( t \)
rigid reminder: at point \( y \) and time \( \tau \)

\[ \delta(x,t) \rightarrow \delta(y,\tau) \]

§2.2 \[ t_r = t - \frac{1}{c} \left| x - \frac{y}{c} \right| \]

\[ t_r = t - \frac{1}{c} \left| x - \frac{y}{c} \right| \text{ implicit eq. for } t_r \]

\[ t_r = t - \frac{v}{c} + \frac{1}{c} x \cdot \frac{v}{c} R(t_r) + O(1/c) \]

\[ \approx t - \frac{v}{c} \quad \text{for } v \ll c \]

\[ = t_e \]
\[
\begin{align*}
\frac{dP}{dt} &= \frac{1}{\sqrt{c^2}} \left( \frac{\hat{x} \cdot \int \frac{d\sigma}{d\omega} \, d\omega \, \delta(\hat{x} \cdot \hat{a})}{\frac{\hat{x}}{c^2}} \right)^2 \\
\end{align*}
\]

**Theorem:** The power radiated by the source per solid angle is

\[
\frac{dP}{d\Omega} = \frac{1}{\sqrt{c^2}} \left( \int \frac{d\sigma}{d\omega} \, d\omega \, \delta(\hat{x} \cdot \hat{a}) \right)^2
\]

**Work:** (1) Power x fields \^ 2 = fields x 1/r

\[
\rightarrow \text{Knowing power per solid angle are outgoing for only the source!}
\]

**Work:** The total power radiated is:

\[
P = \int d\Omega \, \frac{dP}{d\Omega}
\]

### 1.2 Radiating by an accelerated charged point particle

- Where a point particle will drag e that moves with velocity \( v \) to a trajectory \( \hat{R}(t) \).

\[
\frac{d}{dt} \delta(\hat{r} - \hat{R}(t))
\]

**Work:** (2) The not defined "time of emission" \( t_e \) is an approximate expression for the retarded time \( t_r \). \( t_e \)'s valid for \( v \ll c \).

\[
\int d\omega d\hat{a} \, \delta(\hat{x} \cdot \hat{a}) = \frac{1}{\sqrt{c^2}} \int d\sigma \, \delta(\hat{x} \cdot \hat{v} - \hat{v}(t_e) \delta(\hat{x} - \hat{R}(t_e)))
\]

\[
= e \left( \frac{d\hat{v}}{dt} \right)_{\hat{x} \cdot \hat{v}} \delta(\hat{x} \cdot \hat{v}(t_e))
\]
\[
(\mathbf{x} \cdot \mathbf{v})^2 = (t - \mathbf{x} \cdot \mathbf{v})^2 = (v^2)^2 - (\mathbf{x} \cdot \mathbf{v})^2
\]

\[
\frac{dP}{dR} = \frac{1}{mc^2} e \left[ \frac{v^2}{c^2} (\mathbf{v} \cdot \mathbf{x}) - (\mathbf{v} \cdot \mathbf{x})^2 \right] \tag{7}
\]

\( \mathbf{x} \) is the angle between the electron and timevector at rest as seen by the observer.

\[
(\mathbf{x} \cdot \mathbf{v})^2 = (\mathbf{v} \cdot \mathbf{v})^2 - \mathbf{v}^2
\]

\[
\frac{dP}{dR} = \frac{e}{mc^2} (\mathbf{v} \cdot \mathbf{x})^2 \tag{8}
\]

\begin{equation}
\mathcal{P} = \frac{2e^2}{2c^2} (\mathbf{v} \cdot \mathbf{x})^2 \tag{9}
\end{equation}

for \( v \ll c \)

Proof:
\[
\mathcal{P} = \int dR \mathcal{P} = 2 \mathcal{P} \left[ \int d\mathbf{x} \right] (1 - \mathbf{v}^2) = 4 \mathcal{P} \left[ 1 - \frac{v^2}{c^2} \right] = \frac{8\mathcal{P}}{2}
\]

\[
\mathcal{P} = \frac{2e^2}{2c^2} (\mathbf{v} \cdot \mathbf{x})^2
\]

Remarks:
1. This result is sometimes called the Larmor formula. It is valid for nonrelativistic particles.
2. This is the physics behind synchrotron radiation, due to Pauli.
3. It implies that a charged electron (charged electron is bonded motion and a proton) cannot be stable in Problem 28, 29.
New under a guise of many many \( v \ll c \) delays but is still well respond to \( r \approx r_0 + \epsilon \) 

**Proposition:** In this case the redshift power per solid angle is

\[
\frac{d\Omega}{d\Omega} = \frac{1}{\beta c^2} \left( \frac{\Delta \mathbf{r}}{c} \right)^2 \quad \text{(for } v \ll c \text{)}
\]

where \( \Delta \mathbf{r} \) is the dipole moment of the charge distribution

\[
d(t) = \int d\mathbf{r} \frac{\delta}{\beta c^2} \delta(\mathbf{r} - \mathbf{R}(t)).
\]

\( \mathbf{L} \) is its linear density.

**Remark:** (1) For a point charge, \( \mathbf{L}(\mathbf{r}, t) = e \delta(\mathbf{r} - \mathbf{R}(t)) \)

\[
\Rightarrow \mathbf{L}(t) = e \int d\mathbf{r} \frac{\delta}{\beta c^2} \delta(\mathbf{r} - \mathbf{R}(t)) = e \mathbf{R}(t)
\]

\[
\Rightarrow \frac{d}{dt} \mathbf{L}(t) = e \dot{\mathbf{R}}(t)
\]

\[
\Rightarrow \text{Use again the proposition from § 3.2.}
\]

**Lemma:**

\[
\frac{d}{dt} \mathbf{L}(t) = \int d\mathbf{r} \frac{\delta}{\beta c^2} \delta(\mathbf{r}, t)
\]

**Proof:** Charge vanishes \( \Rightarrow \mathbf{L} + \mathbf{v} \cdot \mathbf{L} = 0 \)

\[
\Rightarrow 0 = \int d\mathbf{r} \left[ \mathbf{v} \cdot \mathbf{L}(\mathbf{r}, t) + \mathbf{v} \cdot \frac{\partial}{\partial t} \delta(\mathbf{r}, t) \right]
\]

\[
= \int d\mathbf{r} \left[ \mathbf{v} \cdot \frac{\partial}{\partial t} \delta(\mathbf{r}, t) - \mathbf{v} \cdot \frac{\partial}{\partial t} \delta(\mathbf{r}, t) \right]
\]

\[
= 0 \text{ if } \mathbf{v} \text{ is off just right } c t \in \infty
\]

\[
= -\int d\mathbf{r} \frac{\partial}{\partial t} \delta(\mathbf{r}, t) + \frac{\partial}{\partial t} \int d\mathbf{r} \frac{\delta}{\beta c^2} \delta(\mathbf{r}, t)
\]

\[
= -\int d\mathbf{r} \frac{\partial}{\partial t} \delta(\mathbf{r}, t) + \frac{\partial}{\partial t} \mathbf{L}(t)
\]
\[
\frac{d^2 F}{dt^2} = \left( \hat{x} \cdot \frac{d}{dt} \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, t) \right) \times \frac{d}{dt} \left( \hat{x} \times \frac{d}{dt} \mathbf{a}(t) \right) - \frac{d}{dt} \left( \hat{x} \times \frac{d}{dt} \mathbf{a}(t) \right)
\]

**Remark:** (2) This contribution to the radiative field is called electric dipole radiation.

Now write to the approximative term:

\[
\frac{d^2 F}{dt^2} \text{ is determined by}
\]

\[
\int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, t) = \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, t - \frac{r}{c} + \frac{1}{c} \hat{x} \cdot \mathbf{v})
\]

\[
= \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, t) + \frac{1}{c} \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, \mathbf{v}) \left( \frac{1}{c} \hat{x} \cdot \mathbf{v} \right) \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, t) + \ldots
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) + \frac{1}{c} \frac{\partial}{\partial t} \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, \mathbf{v}) \left( \frac{1}{c} \hat{x} \cdot \mathbf{v} \right)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \frac{1}{c} \frac{\partial}{\partial t} \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, \mathbf{v}) \left( \frac{1}{c} \hat{x} \cdot \mathbf{v} \right) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \frac{1}{c} \frac{\partial}{\partial t} \int d^3 \mathbf{r} \mathcal{E}(\mathbf{r}, \mathbf{v}) \left( \hat{x} \times \mathbf{v} \right) + \text{another term}
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \frac{\partial}{\partial t} \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
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\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

\[
\mathbf{v} \times \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{v}(\mathbf{r}, t)
\]

\[
= \frac{\partial}{\partial t} \mathcal{E}(\mathbf{r}, t) - \hat{x} \times \mathbf{E}(\mathbf{r}, t) + \text{another term}
\]

**Remark:** (2) This is the obvious solution from the problem of determining the electric field due to a time-varying magnetic dipole.
\[ \frac{dP}{dR} = \frac{1}{4\pi c^2} \left[ \hat{x} \times \left( \hat{r}' - \hat{r} \times \hat{r}' \right) \right]^2 \]

will yield in the electric and magnetic dipole moments of the form.

\[ P = \frac{c}{2} \left[ (\hat{a})^2 + (\hat{\omega})^2 \right] \]

**Problem 17**

**Summary**

\[ \text{Proof:} \quad \int dR \left( \hat{x} \times \hat{e} \right)^2 = 2\pi \int d\ell (1-r^2) \hat{e}^2 = 4\pi (1-r^2) \hat{e} = \frac{p_0}{r} \hat{e} \]

\[ \int dR \left( \hat{x} \times (2\hat{x} \hat{e}) \right)^2 = \int dR \left( 2(\hat{x} \cdot \hat{e}) - \hat{e} \right)^2 \]

\[ = \int dR \left[ \hat{e}^2 - 2\hat{e} \cdot \hat{e} + \hat{e}^2 \right] = 2\pi \int d\ell (1-r^2) \hat{e} = \frac{p_0}{r} \hat{e} \]

\[ \int dR \left( \hat{x} \hat{e} \right)^2 + \left( \hat{x} \times (2\hat{x} \hat{e}) \right) = 0 \text{ since linear in } \hat{x} \]

**Remark:** (4) For other 

\[ \text{Problem 17} \]

**Rotational dipole**

\[ \text{Proof:} \quad \int dS \left( \hat{y} \hat{z} + \hat{z} \hat{y} \right) = - \int dS \left( \hat{y} \hat{z} + \hat{z} \hat{y} \right) \]

\[ = \frac{1}{a} \int dS \left( \hat{y} \hat{z} + \hat{z} \hat{y} \right) \]

will yield the quadrupole moment.

\( \text{The contribution to } P \text{ from the line is of } O \left( \frac{r}{c} \right)^2 \)

(5) The magnetic dipole must lose a \( \frac{e}{mc} \) in its definition.

\( \text{The magnetic dipole is electric quadrupole radiating} \)

in the same order as \( \frac{r}{c} \) at least only by considering together, the LC \( \frac{r}{c} \).
Spectral distribution of redshifted energy

In §1 we calculated the total power redshifted by a time-dependent mean redshift: how is the energy distributed over different frequencies?

### 4.1 Retarded potentials $q(x, t)$

\[ q(x, t) = \int d\omega \frac{1}{1 - \frac{\omega}{c}} \mathcal{L} \left( \frac{c}{\omega} (t - \frac{x - \omega t}{c}) \right) \]

**Definition of retarded Fourier transform (cf. §2.2)**

\[
\tilde{f}(\tilde{x}, \tilde{t}) = \int dt e^{i \tilde{\omega} t} f(\tilde{x}, t) \\
\tilde{f}(\tilde{x}, \tilde{t}) = \int \frac{d\omega}{2\pi} e^{-i \omega \tilde{t}} \tilde{f}(\tilde{x}, \tilde{t})
\]

\[
q(\tilde{x}, \tilde{t}) = \int d\omega e^{i \tilde{\omega} \tilde{t}} \int d\omega' \frac{1}{1 - \frac{\omega'}{c}} \mathcal{L} \left( \frac{c}{\omega'} (\tilde{t} - \frac{\tilde{x} - \omega' \tilde{t}}{c}) \right)
\]

\[
= \int d\omega \frac{1}{1 - \frac{\omega}{c}} \int \frac{d\omega'}{2\pi} \mathcal{L} \left( \frac{c}{\omega'} \left( \frac{\omega}{c} - \frac{\omega'}{c} \right) \right) e^{i \omega \tilde{t} - i \omega' \tilde{t}}
\]

\[
= \int d\omega \frac{1}{1 - \frac{\omega}{c}} e^{i \omega \tilde{t}} \mathcal{L} (\tilde{\omega}, \tilde{t})
\]

**Properties:**

The retarded potentials $q(x, t)$ are given by

<table>
<thead>
<tr>
<th>$q(x, t)$</th>
<th>$\tilde{q}(\tilde{x}, \tilde{t})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ q(x, t) = \int d\omega \frac{1}{1 - \frac{\omega}{c}} e^{i \omega \tilde{t}} \mathcal{L} (\tilde{\omega}, \tilde{t}) ]</td>
<td>[ \tilde{q}(\tilde{x}, \tilde{t}) = \int d\omega \frac{1}{1 - \frac{\omega}{c}} e^{i \omega \tilde{t}} \mathcal{L} (\tilde{\omega}, \tilde{t}) ]</td>
</tr>
</tbody>
</table>

where $\mathcal{L}(\tilde{\omega}, \tilde{t})$ and $\mathcal{L}(\tilde{\omega}, \tilde{t})$ are the retarded Fourier transforms of the charge and current densities. 
4.2 Asymptotic polarized fields

For large distances $r = |x|$ from the source, the expression for $\phi$ applies:

$$\phi(x, u) = \int \frac{r}{2} e^{i \omega t} \left[ \frac{1}{1 + i \omega} \right] e^{i r^2 / (2 \epsilon^2)} \tilde{g}(x, u)$$

$$= \frac{1}{r} e^{i \omega t} \int \frac{r}{2} e^{-i \omega \cdot \tilde{x} \cdot x} \tilde{g}(x, u) + O(\|r\|^2)$$

**Def.**

$$\tilde{x} = \frac{u}{c} \hat{y}$$

is called a wave vector.

**Remark.**

1. This is quite similar to §1.5, remark (1).

2. For large $|x|$ from the source, the wave equation applies:

$$\phi(x, u) = \frac{1}{r} e^{i \omega t} \int \frac{r}{2} e^{-i \omega \cdot \tilde{x} \cdot x} \tilde{g}(x, u)$$

$$= \frac{1}{r} e^{i \omega t} \tilde{g}(x, u)$$

with $\tilde{g}(x, u)$ the spherical Fourier

**Analysis.**

$$\tilde{A}(x, u) = \frac{1}{r} e^{i \omega t} \frac{1}{i} \tilde{g}(x, u)$$

**Proposition:** For large $|x|$ from the source, the fields are given by

$$\tilde{A}(x, u) \simeq i \frac{u}{c} \frac{e^{i \omega t}}{r} \tilde{g}(x, u)$$

$$\tilde{E}(x, u) \simeq -\tilde{x} \cdot \tilde{A}(x, u)$$

**Remark.**

1. The expression for $\tilde{E}$ is the of $\tilde{A}$ follows directly from the proposition in §1.1.
Proof: \[ \vec{\Pi}(x_1, u) = \nabla \times A(x_1, u) \]
\[ \Rightarrow \vec{E}(x_1, u) = E_{\text{ext}} \nabla \times A(x_1, u) \]
\[ = E_{\text{ext}} \left( \frac{\partial}{\partial r} \frac{1}{r} \frac{e^{\text{i}kr}}{\text{i}} \vec{e}_r \right) \hat{\omega}(x_1, u) \]
\[ = E_{\text{ext}} \frac{\partial}{\partial r} \frac{1}{r} \frac{e^{\text{i}kr}}{\text{i}} \vec{e}_r \cdot \hat{\omega}(x_1, u) \]
\[ = E_{\text{ext}} \frac{1}{r} \frac{e^{\text{i}kr}}{\text{i}} \vec{e}_r \cdot \hat{\omega}(x_1, u) = 0 \text{(for r)} \]
\[ \frac{\partial}{\partial r} \frac{1}{r} e^{\text{i}kr} = \frac{e^{\text{i}kr}}{r} \frac{\partial}{\partial r} \frac{1}{r} \]
\[ = i k r \frac{e^{\text{i}kr}}{r} \]
\[ = i k r \frac{e^{\text{i}kr}}{r} \]
\[ \Rightarrow \hat{E}(x_1, u) = 0 \]

\[ \Rightarrow \hat{E}(x_1, u) = 0 \text{ for } r \text{, follow from §2.1 proof. in model (2)} \]

4.2 The spectral distribution of the radiated energy

Known: The total energy radiated by the source per solid angle at \( r \) is

\[ \frac{dU}{dRdu} = \frac{u^2}{4\mu c^2} |\hat{\omega} \cdot \hat{\omega}(\hat{u}, u)|^2 \]

Remark: (1) Under a static source: \( \hat{\omega}(\hat{u}, t) = \hat{\omega}(\hat{u}, \hat{u}) \Rightarrow \hat{\omega}(\hat{u}, u) = \delta(\hat{u}) \)

Proof: The instantaneous flux of energy is given by the Poynting vector.

\[ \Delta t \approx 3.6 \]

\[ \hat{P}(x_1, t) = \frac{u^2}{4\mu} \hat{E}(x_1, t) \times \hat{B}(x_1, t) \]

\[ \Rightarrow \text{The total energy radiated into a solid angle dR at} \]

\[ \Delta t \approx 3.6 \]

\[ \frac{dU}{du} = \oint_{dt} R^2 \hat{N} \cdot \hat{P}(x_1, t) \]
\[ \frac{dW}{d\mu d\omega} = \frac{\mu^2}{2\pi^2} |\hat{\mathbf{r}}^2(x', u')|^2 \]

4.4 Spectral distribution for dipole radiation

\[ \frac{dW}{d\mu d\omega} \text{ is given by the Fermi's rule of the unit dipole} \]

\[ \frac{dW}{d\mu d\omega} \text{ where } \lambda = k\lambda = \frac{\omega}{c} = \frac{\mu}{\lambda} \]

will be the wavelength of the radiation.

Under small waves i.e., \( |\hat{\mathbf{r}}| \ll 2 \lambda \).

Example: (1) For an electron radiation without light, we have

\[ |\hat{\mathbf{r}}| \ll c \mu \omega \]

\[ \lambda \approx \text{hundreds of } \mu \]
\[
\frac{d}{dt} \frac{\partial}{\partial u} = \left[ 1 - i \cdot \Delta \cdot \cdots \right] \int d\theta \text{e}^\text{i \omega t} \theta (\sigma, t)
\]
\[
= \text{int} \cdot \frac{d}{dt} \theta (\sigma, t) + O(\alpha / \beta) \quad \text{will be linear}
\]
\[
\text{line}
\]
\[
= \text{int} \cdot \frac{d}{dt} \theta (\sigma, t) + O(\alpha / \beta)
\]
\[
= - \omega \cdot \theta (\sigma, t) + O(\alpha / \beta)
\]

**Proposition:**

If \( a \) is the linear dimension of the wave, \( \alpha \) is the wavelength of the redshifting, \( \gamma \) is the boost factor, \( \epsilon \) is the energy redshifted per unit, then energy redshifted unit frequency is

\[
\frac{d\theta}{d\varepsilon} = \frac{\Delta \cdot \varepsilon}{\gamma^2 c^2} \frac{1}{\theta (\sigma, t)^2}
\]

where \( \theta \) is the angle between \( \theta \cdot \sigma \), and \( \theta (\sigma, t) \) is the hyperboloidal form in terms of \( \sigma (t) \).

**Proof:**

\[
\text{\textit{Approxi}}
\]

\[
\text{\textit{makely}}
\]

\[
\text{\textit{the total redshift energy per frequency is}}
\]

\[
\frac{d\theta}{d\varepsilon} = \frac{\Delta \cdot \varepsilon}{\gamma^2 c^2} \theta (\sigma, t)^2
\]

\[
\text{\textit{proof:}} \quad \int d\varepsilon \int d\theta \frac{d\theta}{d\varepsilon} = 2 \int d\varepsilon \frac{d\theta}{d\varepsilon}
\]

\[
\int d\varepsilon \int d\theta \frac{d\theta}{d\varepsilon} = 2 \int d\varepsilon \frac{d\theta}{d\varepsilon} = \frac{d\theta}{d\varepsilon}
\]

\[
\text{\textit{ex:}} \quad \text{A point charge will trajectory } \frac{d\theta}{d\varepsilon} \text{ and velocity } \frac{d\theta}{d\varepsilon}
\]

\[
\to \frac{d\theta}{d\varepsilon} = \frac{d\theta}{d\varepsilon} \theta (\sigma, t) \theta (\sigma, t)
\]

\[
\text{\textit{example:}} \quad \text{A point charge will trajectory } \frac{d\theta}{d\varepsilon} \text{ and velocity } \frac{d\theta}{d\varepsilon}
\]

\[
\to \frac{d\theta}{d\varepsilon} = \frac{d\theta}{d\varepsilon} \theta (\sigma, t) \theta (\sigma, t)
\]
Either the x-axis or the y-axis must be in the spherical plane. The waist must be in the 1st or 2nd quadrant, but not in the 3rd!

Now modelled will yield:

\( (\omega \cdot \hat{\ell}, \hat{\nu} \cdot \hat{\ell}, \hat{\nu} \cdot \hat{\nu}) \) \( (0 \leq \ell < \pi) \)

permetting the vertex of constant \( \hat{\ell} \), or, equivalently,

\( (\hat{\nu} \cdot \hat{\ell}, \hat{\nu} \cdot \hat{\ell}, \hat{\nu} \cdot \hat{\nu}) \)

Now using

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} = \left| \frac{\partial u}{\partial r} \right| \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

\[ \frac{\partial u}{\partial \omega} \times \frac{\partial u}{\partial \theta} \times \frac{\partial u}{\partial r} \]

No reduction in the direction of \( \hat{\nu} \)

If the reduction, then isotropic, the polar diagram will be a circle, which is permuted by

\[ (x, y) = (\omega \cdot \hat{\ell}, \omega \cdot \hat{\nu}) \] \( (0 \leq \ell < \pi) \)

Now this gets modified by \( \hat{\nu} \)

\[ (x, y) = (\omega \cdot \hat{\ell}, \omega \cdot \hat{\nu}) \]

will \( \ell = 0 \) correspond to the \( \hat{\ell} \)-circle.

For \( \ell \rightarrow 0 \), \( x \rightarrow 0 \), \( y \rightarrow 0 \) \( x \) \( \hat{\ell} \)-circle.

Now we reach out of the orbital plane. For isotropic reduction, the vertex of constant \( \hat{\ell} \) would be a sphere:

\[ (\omega \cdot \hat{\ell}, \omega \cdot \hat{\nu}, \omega \cdot \hat{\nu}) \] \( (0 \leq \ell < \pi, 0 \leq \varphi < \pi) \)

We can rotate the sphere out of the plane.
\[ \frac{d}{dt}(1) = \frac{d}{dt}(\Sigma \mathbf{v}(t)^2) \implies \dot{\mathbf{v}} = \mathbf{v}(t) \]

\[ \frac{d}{dt} \left( \frac{2}{3} \mathbf{v}^2 \right) = \frac{2}{3} \mathbf{v} \cdot \dot{\mathbf{v}} \]

**Remark:**
1. \( \ddot{u} = u \) is given by the hyperbolic function of the osculating
2. \( \ddot{t} \) is constant with the same function, \( \ddot{t} = \mathbf{v}(t) \)

**Example:**
1. Suppose the velocity vector is constant \( \mathbf{v}(t) \)
   - in a circle \( \implies \dot{\mathbf{v}} = \mathbf{v} \) is purely radial
   - \( v \) is moving radially

\[ v \] No radiation is emitted from the direction of \( v \) \( \dot{v} \)
- in the orbit plane, the radiation is only in a 2-local sphere

\[ v \]

\[ v \] in 2-d it looks like a torus: \( \mathbb{R}^2 \) \( \mathbb{R}^1 \)
Example: Reducing by a damped harmonic oscillator

with a damped particle in a harmonic potential (oscillating spring $v_0$) will dampen $x(t)$ as $t$ increases. Eq. of motion:

$$\ddot{x} = -v_0^2 \dot{x} - \dot{x}$$

Remark: (1) The term of $x(t)$ refers to the harmonic oscillator.

(2) This is a useful model for a damped system in an oscillating

initial conditions: $y(t=0) = 0$, $\dot{y}(t=0) = 0$

where: For small damping $\gamma < v_0$, the solution of (1) is

$$y(t) = a\cos\omega t e^{-\gamma t/2} \quad (t > 0) \quad \text{pro: complex form as Problem 54}$$

Thus, the solution is

$$\dot{y}(t) = -a\omega \sin\omega t e^{-\gamma t/2} \left[1 + 0(\gamma/v_0)\right]$$

with Fourier slant $y(t) = -a\omega \int_0^t \cos\omega t \sin\omega t e^{-\gamma t/2}$

$$\frac{a\omega}{\gamma} \int_0^t \left(e^{\gamma t/2} \cos\omega t - e^{\gamma t/2} \sin\omega t\right)$$

$$= -\frac{a\omega}{\gamma} \left[\frac{\gamma}{(u+u_0) - \gamma t} - \frac{\gamma}{(u-u_0) - \gamma t}\right]$$

$$= \frac{a\omega}{\gamma} \left[\frac{\gamma}{u+u_0 + i\gamma t} + \frac{\gamma}{u-u_0 + i\gamma t}\right]$$

$$= \frac{a\omega}{\gamma} \left[\frac{1}{u-u_0 + i\gamma t} - \frac{1}{u+u_0 + i\gamma t}\right]$$

let $\omega > 0$ (discriminant for $\omega < 0$ is complex)

$\Rightarrow y(t)$ is dominated by the first term for $\omega = v_0$

$$|y(t)|^2 \approx \frac{e^{2v_0^2}}{4} \frac{1}{(u-u_0)^2 + \gamma^2}$$

$$\frac{dy}{du} \approx \frac{2e^2}{3\pi^2} \frac{e^{2v_0^2}}{4} \frac{u^2}{(u-u_0)^2 + \gamma^2}$$
\[
\frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \frac{1}{(u-u_0)^2 + \gamma^2} \quad \text{for } u = u_0
\]

\text{(1) Frequency is a}

\text{limiting width

about } u = u_0 \text{ will}

\text{yield } \gamma.

\text{(2) Under the total energy}

radiated:}

\[
U = \int_0^\infty \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \frac{1}{(u-u_0)^2 + \gamma^2} \, du = \frac{\gamma}{2} \int_0^\infty \frac{dx}{x^2 + 1} = \frac{\gamma}{2} \int_0^\infty \frac{dx}{x^2 + 1} = \frac{\gamma}{2} \int_0^\infty e^{-x} dx = \frac{\gamma}{2}
\]

\text{ed upon with the initial energy of the oscillator:}

\[
t_0 \gamma = \frac{\gamma}{2} \int_0^\infty e^{-x} dx = \frac{\gamma}{2} \int_0^\infty e^{-x} dx = \frac{\gamma}{2}
\]

\Rightarrow \quad U = t_0 \gamma \frac{\gamma}{2} \frac{2}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma \frac{t_0 \gamma}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma
\]

\text{but the oscillator energy } t_0 \gamma \text{ must have gone into the radiation energy } U \Rightarrow \quad U = \frac{1}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma \frac{t_0 \gamma}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma
\]

\text{(3) Upon with Path 19, which conclude that radiation power}

\text{ed concluded } U_\text{rad} = t_0 \gamma \frac{2}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma \frac{t_0 \gamma}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma \frac{t_0 \gamma}{2} \frac{e^{\frac{1}{2}w_0^4}}{b^2 c^2} \gamma
\]

\text{as concluded above.} \Rightarrow \text{The two approaches are weight}

\text{(4) Path 44 for a non uniform distribution of the}

\text{approximation made above.}
5.1 The time-Dyson function, or the microscopic part, is:

\[
\frac{d^2u}{dudv} = \frac{\omega^2}{\nu^2 c^2} \left| \hat{X} \hat{J} (\hat{A}, u) \right|^2
\]

\[
= \frac{\omega^2}{\nu^2 c^2} \left( \hat{X} \int dt e^{iut} \hat{J} (\hat{A}, t) \right) \left( \hat{X} \int dt' e^{-iut'} \hat{J} (\hat{A}, t') \right)
\]

\[
= \frac{\omega^2}{\nu^2 c^2} E \hat{X} \hat{J} \hat{X} \hat{J} \int dt dt' e^{i(u-t-t')} \int d(\hat{A}, t) d(\hat{A}, t')
\]


\[
\int dt dt' \hat{J} (\hat{A}, t) \hat{J} (\hat{A}, t') e^{i(u-t-t')} = \begin{cases} 1 & t = \hat{A} + \hat{A}c \hat{F} \\ t' = \hat{A} - \hat{A}c \hat{F} \end{cases}
\]

\[
= \int dt \int dt' e^{i\omega t} \hat{J} (\hat{A}, t+\hat{A}c \hat{F}) \hat{J} (\hat{A}, t-\hat{A}c \hat{F})
\]

\[
= \int dt \int dt' e^{i\omega t} W_{\hat{A}} (\hat{A}, \hat{A}c \hat{F}, \hat{A}c \hat{F})
\]

where \( W_{\hat{A}} (\hat{A}, \hat{A}c \hat{F}, \hat{A}c \hat{F}) = \int d(\hat{A}, t+\hat{A}c \hat{F}) d(\hat{A}, t-\hat{A}c \hat{F}) \)

**Remark:**

1. \( W_{\hat{A}} \) is an example of what is called a (time) Dyson function. It appears here twice as a 'correlation' or 'microscopic' time \( \hat{A} \).

2. Only when times \( \hat{A}c \hat{F} \) will actually contribute to the \( \hat{A}c \) time, whereas all times \( \hat{A}c \hat{F} \) will contribute to the \( \hat{A}c \) time.
(2) This means, we if the two time scales are well
apart. E.g., a lower path of duration $T \gg 1/\omega$

\[ \frac{d^2 \mathcal{P}(t)}{du d\nu} = \frac{\omega^2}{4\pi^2} \xi_j \xi_k \xi_l \chi \gamma(n) \phi(n) e^{i\omega \nu} W_{\omega \nu}(\nu, \omega) \]

is called the incoherent power spectrum.

Remark: (1) The spectral distribution of the redshift energy is

\[ \frac{d^2 \mathcal{P}(t)}{du d\nu} = \int d\nu \frac{d^2 \mathcal{P}(t)}{du d\nu} \]

\section{Cerenkov radiation}

With a point particle as in § 3.2:

\[ \mathcal{F}(\xi, t) = e^{i\nu t} \delta(\xi - \nu \mathcal{R}(t)) \]

and suppose it moves with velocity $v$ along a straight trajectory:

\[ \mathcal{R}(t) = \frac{v}{c} t, \quad \mathcal{F}(t) \equiv \mathcal{F} = \omega \mathcal{C} \]

Remark: (1) The known fact in vacuum, this will not result in

radiation:

\[ \rightarrow \mathcal{F}(\xi, t) = \int \frac{d^3 k}{(2\pi)^3} e^{-i\mathbf{k} \cdot \mathbf{v}} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} = e^{i\omega t} e^{-i\mathbf{k} \cdot \mathbf{x} / c} \]

\[ \mathcal{F} = \epsilon \mathcal{C} \]

\[ \rightarrow W_{\omega \nu}(\nu, \omega) = \frac{\mathcal{F}(\xi, t)}{i \omega} e^{-i\mathbf{k} \cdot \mathbf{x} / c} = \mathcal{E}_\nu e^{-i\mathbf{k} \cdot \mathbf{x} / c} \]

\[ W_{\omega \nu}(\nu, \omega) = \mathcal{E}_\nu e^{-i\mathbf{k} \cdot \mathbf{x} / c} \]
Remark: (2) The Dijson field is independent of $t$, as one would expect for uniform motion.

\[ \frac{d^2 E(t)}{dt^2} = \frac{\omega^2 E_i}{4 \pi \epsilon} \left[ \delta_{i\lambda} \delta_{\lambda\nu} \frac{\partial^2}{\partial x_i \partial x_\nu} \right] \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_\nu} e^{-i \mathbf{k} \cdot \mathbf{x} - i \epsilon \omega (\mathbf{k}/c)^2} \]

\[ = \frac{\omega^2 E_i}{4 \pi \epsilon} \left( \frac{\epsilon}{c} \right)^2 \omega \delta(\omega (1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2} \omega)) \]

\[ = \frac{\omega^2 E_i}{4 \pi \epsilon} \left( \frac{\epsilon}{c} \right)^2 \omega \delta(\omega (1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2} \omega)) \]

Remark: (2) $v/c < 1$, $\omega d < v \rightarrow 1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2} \omega > 0$

$\rightarrow$ no radiation, i.e. expansion will stop unless $\nu/c > \text{speed of light}$

(1) in water, $c \rightarrow c/n$ will be the index of refraction.

(5) Strictly speaking, this requires a thing of electromagnetic waves in continuous matter. Then we assume that $c \rightarrow c/n$ $\mathbf{k}$ still has to be used in wave vector. Also keep in mind: We can apply a nondeterministic approximation to a non-Newtonian $\nu/c$ is no longer valid (see Problem 45).

(6) $\nu$ is strongly dependent, so we should look at $\nu(0)$.
The total radiated power is:

\[ P = \frac{dE}{dt} = \int_0^\infty \frac{dP}{d\nu} \, d\nu = \frac{2e^2}{\pi c^2} \int_0^\infty \frac{\nu^4}{(\nu^2 + \frac{v^2}{c^2})^2} \left(1 - \frac{v^2}{\nu^2 + \frac{v^2}{c^2}}\right) \Theta \left(\frac{v^2}{\nu^2 + \frac{v^2}{c^2}} < \frac{c}{\nu}\right) \, d\nu \]

\[ = \frac{e^2}{\pi c^2} \int_0^\infty \frac{\nu^4}{\nu^2 (\nu^2 + \frac{v^2}{c^2})^2} \left(1 - \frac{v^2}{\nu^2 + \frac{v^2}{c^2}}\right) \Theta \left(\frac{v^2}{\nu^2 + \frac{v^2}{c^2}} < \frac{c}{\nu}\right) \, d\nu \]

Remark: (9) The line in use has photons, each photon has an energy \( E \). The number of photons per range \( d\lambda \) is

\[ \frac{dN}{d\nu d\nu} = \frac{\nu^4}{\pi c^2} \left(1 - \frac{v^2}{\nu^2 + \frac{v^2}{c^2}}\right) \]

This is the first step constant.

A typical rector electron beam at \( v \approx 0.9c \) at \( n(\nu) \) for water decreases exponentially with the energy. The peak of the beam, at \( n(\nu) \approx 0.91c \approx 3.0 eV \), is the bremsstrahlung observed in a water-cooled reactor, favors above.
\[ \frac{d^2 \mathcal{P} \left( \nu \right)}{d\nu \, d\Omega} = \frac{\left( 1 + \frac{\nu^2}{c^2} \right) \nu \, \delta \left( 1 - \frac{\nu_0}{c} \nu \right)}{\omega^2 \left( \frac{\omega}{c} \right)^2} \]

**Problem 45**: A particle moves through a medium at a speed less than the speed of light in that medium with redshift. The total power is:

\[ \mathcal{P} = \int d\Omega \frac{d^2 \mathcal{P}}{d\nu \, d\Omega} = \frac{1}{\omega^2} \frac{\omega_0}{c^2} \left( 1 - \frac{c^2}{\omega^2} \right) \int d\nu \left( 1 - \frac{\nu_0}{c} \nu \right) \delta \left( 1 - \frac{\nu_0}{c} \nu \right) \]

**Remark**:

1. This is nonzero only for the (finite) frequency range when \( \nu < \frac{\omega_0}{c} \).
2. The total redshifted power
   \[ \mathcal{P} = \int d\nu \, d\Omega \frac{d^2 \mathcal{P}}{d\nu \, d\Omega} \]
   is finite!
3. This is the redshifted energy per unit solid angle for a source moving with a speed
   \( \nu \) through a medium, as observed by an observer moving with a speed \( \nu_0 \).
Fig. 7 Segelstein's values for the real part of the refractive index of water for wavelengths from 10 nm to 10 m
Here is the index or refraction of water as a function of the wavelength (from http://www1.lsbu.ac.uk/water/dielectric_constant.html#refract)

\[
\frac{dn}{d\nu} |_\text{blue} = \frac{1 - \frac{c}{v^2 n_i}}{1 - \frac{c}{v^2 n_d}} |_\text{red} = 1.07
\]

n in the blue is about 1.35, and n in the red is about 1.33, and \((1.35/1.33)^2 = 1.03\). So the frequency dependence of the index of refraction favors the blue end of the visible spectrum, but this is a rather small effect.

A larger effect is the frequency dependence of the absorption coefficient:

And here is the absorption coefficient (from http://www1.lsbu.ac.uk/water/water_vibrational_spectrum.html)

Assume that the Čerenkov photons run through 1m of water before emerging into air. Then virtually all of the blue photons will make it, but only about \(1/e = 0.37\) of the red ones do!

The visible and UV spectra of liquid water

Conclusion: The chief reason for the blue color of the Čerenkov radiation is the frequency dependence of the absorption coefficient of water. The frequency dependence of the index of refraction also favors the blue end of the spectrum, but that’s a much smaller effect.
6.1 Relativistic motion of a charged particle in a homogenous $A$-field

\[ \frac{d\vec{p}}{dt} = \frac{e}{c} \vec{E} \times \vec{A} \] \hspace{1cm} (8)

will $\vec{p} = \gamma \vec{p}^0$ the momentum ($\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$)

Remark: (1) (8) is true for both relativistic and non-relativistic motion.

(2) Form is purely formal $\Rightarrow E = \frac{p}{m} \Rightarrow c \gamma E = \frac{p^0}{c}$ will $E$ in particle's energy.

$\Rightarrow$ The eq. of motion can be with

\[ \frac{E}{c} \frac{d\vec{v}}{dt} = \frac{e}{c} \vec{E} \times \vec{A} \hspace{1cm} \Rightarrow \frac{d\vec{v}}{dt} = \frac{e}{E} \vec{E} \times \vec{A} = -\frac{e}{E} \vec{A} \times \vec{v} \]

def.: $\omega_0 = \frac{1}{mc^2} \frac{E}{e}$ is called Larmor frequency

Remark: (2) For non-relativistic particles, $\omega_0 = \frac{1}{mc} \frac{E}{mc} \approx \frac{1}{mc} \omega_0$ is called cyclotron frequency.

Initial condition: $\vec{v}_0 = \vec{v}_0$ for all times.
The particle moves on a circle of radius \( R = \frac{v}{u_0} = \frac{\nu}{c} \frac{E}{mc^2} \).

The momemtum is related to the radius by
\[
p = \frac{E}{c^2} v = \frac{1}{c} l c \text{d}R
\]

Remark: (4) This provides a way bullied for moment of a relativistic particle.

6.2 In pointy space of nuclear radius \( \hat{r} \) inside motion in the \( x-y \)-plane with an observer at point \( x \) and \( d = \frac{d}{\sqrt{(x, y)}} \).

Usual coordinate \( \hat{r} \) for \( t = (x, 0, z) \).

\[
\hat{r} = \begin{pmatrix} i & 0 & 0 \end{pmatrix}
\]

and initial velocities at \( t \): \( \hat{v}(t) = R(wu_0 t, iwu_0 t, 0) \)

\[
\hat{v}(t) = \begin{pmatrix} v(-iwu_0 t, iwu_0 t, 0) \end{pmatrix}
\]

The unit vector \( \hat{z} \)

\[
\hat{z}(t) = e^{-iwx_0 t} e^{-iwy_0 t} e^{iwx_0 t} e^{-iwy_0 t} \hat{z}(t)
\]

\[
\hat{z}(t) = e^{-i\omega x_0 t} e^{-i\omega y_0 t}
\]
The proof yields

\[ \frac{d^2 G(t)}{du \, d\omega} - \frac{\omega^2}{\nu^2 c^2} \left[ du \, e^{i\omega \bar{u}} \left( \begin{array}{c} \frac{\partial}{\partial \omega} \tilde{F}(\bar{u}, \nu \bar{c} \omega) \tilde{F}(\bar{u}, \nu \bar{c} \omega) - \\
- e^{i\omega \bar{u}} \tilde{G}(\bar{u}, \nu \bar{c} \omega) \tilde{G}(\bar{u}, \nu \bar{c} \omega) \end{array} \right) \right] \]

Proof:

\[ f(\omega, t) \rightarrow \text{the angular (vibration) } \text{e}^{i\omega \bar{u}} \text{is} \]

\[ \text{Eig. Ele.} \quad \hat{x} \cdot \hat{x} \text{e}^{i\omega \bar{u}} \text{when} (\bar{x}, \bar{z}, \bar{c}) = \frac{\hat{x}}{\sqrt{\bar{x} \cdot \hat{x}}} \text{and} \] \[ \bar{x} = \hat{x}, \quad \text{ed} \quad \hat{x} = \omega / c \]

\[ \text{ed} \quad \text{the wave number} \quad \bar{c}, \quad \bar{z} \quad \xi \quad \bar{c} \quad \hat{x}, \xi = \frac{\partial}{\partial \xi} \tilde{G}(\bar{x}, \bar{c}) \]

\[ \text{initial} \quad \bar{c} \tilde{G}(\bar{x}, \bar{c}, \bar{c}) = \frac{\partial}{\partial \xi} \tilde{G}(\bar{x}, \bar{c}, \bar{c}) = \tilde{G}(\bar{x}, \bar{c}, \bar{c}) \]

\[ \Rightarrow \quad \tilde{G}(\bar{x}, \bar{c}) = C \tilde{G}(\bar{x}, \bar{c}) \]

\[ \tilde{G}(\bar{x}, \bar{c}, \bar{c}) \cdot \tilde{G}(\bar{x}, \bar{c}, \bar{c}) = \nu^2 \text{e}^{i\omega \bar{u}} \text{u}_{0} \tilde{c} \]

Proof:

\[ \frac{1}{\sqrt{2} \nu} \tilde{G}(\bar{x}, \bar{c}), \tilde{G}(\bar{x}, \bar{c}) = u_{0} (u_{0} \bar{c}) u_{0} (u_{0} \bar{c}) + u_{2} (u_{0} \bar{c}) u_{2} (u_{0} \bar{c}) \]

\[ \left( \omega^2 - \nu^2 \right) - \frac{2 \omega l_0 \bar{c}}{c} \tilde{G}(\bar{x}, \bar{c}, \bar{c}) \]

\[ \frac{2 \omega l_0 \bar{c}}{c} \tilde{G} (\bar{x}, \bar{c}, \bar{c}) = \frac{2 \omega l_0 \bar{c}}{c} \tilde{G} (\bar{x}, \bar{c}, \bar{c}) = \frac{2 \omega l_0 \bar{c}}{c} \tilde{G} (\bar{x}, \bar{c}, \bar{c}) \]

\[ \Rightarrow \quad \tilde{G}(\bar{x}, \bar{c}) = C \tilde{G}(\bar{x}, \bar{c}) \]

\[ \text{via} \quad \tilde{G}_{\nu}(\bar{x}) \rightarrow \text{exact function of the first kind.} \]
\[ \text{Proof:}\quad \text{The desired probability is given by} \quad e^{itw_0} = \sum_{n=-\infty}^{\infty} e^{-iwn_0t} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = -\frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = \frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = -\frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = \frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = -\frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = \frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = -\frac{w_0^2}{v^2} e^{itw_0} \]

\[ \Rightarrow \quad \frac{d^2}{dt^2} e^{itw_0} = \frac{w_0^2}{v^2} e^{itw_0} \]

Remark: (1) For the macroscopic porous sphere we can not introduce a new kinetic energy via the microscopic kinetic scale set by $1/u_0$ as shown over a oscillation period.

\[ \text{since} \quad e^{-i(mn)u_0t} = \delta_{mn} \quad \text{when} \quad f(t) \quad \text{indicates a hump over one oscillation period.} \]

\[ \text{Proof:} \quad e^{-i(mn)u_0t} = \frac{2\pi}{2\pi} \int_0^{2\pi} e^{-i(mn)u_0t} dt = \frac{1}{2\pi} \int_0^{2\pi} e^{-i(mn)x} dx \]

\[ = \frac{1}{2\pi} \int_0^{2\pi} \left[ \cos((mn)x) + i\sin((mn)x) \right] dx = \delta_{mn} \]
\[
\frac{d^2 \tilde{P}}{d \omega dR} = \frac{\omega^3 e}{\gamma c^2} \sum_{m=\infty}^{\infty} \left( \int_{-\gamma}^{\gamma} (\frac{\omega}{c} \text{Re}\, i \mathbf{d} \cdot \mathbf{E})^2 \frac{d\omega}{2\pi} e^{-i(\omega - \omega_0)R} \right) \times \left[ \frac{1}{\gamma} \left( e^{i\omega_0 R} + e^{-i\omega_0 R} \right) - 1 \right]
\]
\[
= \frac{\omega^3 e}{\gamma c^2} \sum_{m=\infty}^{\infty} \left( \int_{-\gamma}^{\gamma} (\frac{\omega}{c} \text{Re}\, i \mathbf{d} \cdot \mathbf{E})^2 \frac{d\omega}{2\pi} \left( \delta(\omega - (m-1)\omega_0) + \delta(\omega - (m+1)\omega_0) \right) \right) \times \delta(\omega - \omega_0)
\]
\[
= \frac{\omega^3 e}{\gamma c^2} \sum_{n=0}^{\infty} \left( \int_{-\gamma}^{\gamma} (\frac{\omega}{c} \text{Re}\, i \mathbf{d} \cdot \mathbf{E})^2 \frac{d\omega}{2\pi} \left( \delta(\omega - (n-1)\omega_0) + \delta(\omega - (n+1)\omega_0) \right) \right) \times \delta(\omega - \omega_0)
\]

\[\text{proof:} \quad \text{The exponent of the Jacobian is } \frac{\omega R}{c} \text{Re}\, i \mathbf{d} \cdot \mathbf{E} \quad \text{and the Jacobian obeys} \quad \delta(\omega - \omega_0) \times \delta(\omega - \omega_0)\]
\[ \frac{dP_x}{P_x} = \frac{dL}{m^{1/2}} \]

The redshift is expected to be some about \( z = \frac{1}{2} \) of opjiu wyne \( z \approx m^{-1/2} \).

Most of the redshift is matched into demographics with \( m = \frac{1}{2} \).

The consequences is \( 4L \times 1/L \) (in Proth 42 for a different report where it leads to the same work).

At the AIC, \( v/c = 0.999999 \Rightarrow z = 1.5 \).

6.2 Anitferi explanation of the main feature

6.2 The antitfera redshift as determined by

(i) a frequent, right side forward direction

(ii) high precision (high precision of the potential, observable phenomena)

Thus the determination can be justified, understood as follows:

For a point position we have the following relation position from Proth 35:

\[ \frac{A(t, x)}{B(t, x)} = \frac{e^{V(t, x)} / c}{|x - x(t, x)|} = \frac{e^{V(t, x)} / c}{|x - x(t, x)|} \]

where \( t_0 = t = \frac{1}{c} |x - x(t, x)| \)

Let \( \phi = \phi(t, x) \), then

\[ \frac{1}{1 - c \phi / c} = \frac{1}{1 - \phi} \frac{c}{1 - \phi} \]

\[ \phi = \frac{c}{1 - \phi} \]

\[ \phi = \frac{c}{1 - \phi} \]

\[ \phi = \frac{c}{1 - \phi} \]

\[ A \text{ is applicable only for } \phi \leq 1/3 \text{. This explains } (3.1) \]
Now consider a particle in a circular orbit.

The light reaching the observer is along a helix as of the orbit given by
\[ \frac{dS}{dt} = \frac{\Phi}{2\pi R} \Rightarrow dS = R \Phi dt \]

The radial line is marked as \( \phi \) along a time interval
\[ \frac{dS}{dt} = \frac{dS}{dt} \Rightarrow dt = \frac{1}{\omega_R} \Phi \]

The typical frequency emitted is
\[ \omega = \frac{1}{dt} = \omega_R / \Phi = \omega_R \Phi \]  

This holds in the rest frame of the particle. From the
observer's point of view, \( dS / dt \) is shorter by a factor of \( \gamma \)
(Compton effect) \( \Rightarrow \omega \times \frac{1}{dt} < \frac{1}{\Phi} \) is larger by a factor
of \( \gamma \). Finally, the observer sees a Doppler shifted
frequency \( (d \omega / dt \xi) \), which provides another factor of \( \gamma \)
(Compton effect)

\[ \omega_\text{observed} = \omega_R \gamma \Phi \gamma = \omega_R \gamma^2 \]

His expression (ii) of Doppler shift

6.4 The potential of hydrogen molecule

Potential of light is measured via the effect of the \( E \)-field.

\[ E_{\text{light}} = -\frac{\Phi}{4\pi} \int \frac{dS}{R^2} \times \cdot \cdot (E(x,u) \times \tilde{q}(x-u)) \]

And from the proper \( \Phi \) do we \( E(x,u) = -x \times \tilde{q}(x,u) \Rightarrow \tilde{q}(x,u) = x \times E(x,u) \)
\[ \mathbf{E}(x,u) \propto x \cdot \hat{x} \cdot (x',u)\]

\[ \Rightarrow \mathbf{E}(x',u) \propto -x \cdot (x',u)\]

Our expression for the proper space remains valid if we replace \( x \rightarrow -x \cdot (x',u) \).

\[ \frac{d^2 \mathcal{P}(t)}{du \, dR} = \frac{\omega^2}{4 \pi c^2} \int d\omega \, e^{i \omega \phi} \left[ -x \cdot (x',u) \right] \cdot \left[ -x \cdot (x',u) \right] \]

\[ = \left[ -x \cdot (x',u) \right] \cdot \left[ -x \cdot (x',u) \right] \]

**Def. 1:** Will a world line which moves as \( \hat{x}(t) \):

- Orbit in \( x-y \) plane, \( \hat{x} = (\text{in}, 0, \text{in}) \),
- Define parallel polarization as \( \hat{E} \parallel \hat{E} \) when
  \[ \hat{E} = (0, \frac{3}{2}, 0) \]
- Define perpendicular polarization as \( \hat{E} \parallel \hat{E} \) when
  \[ \hat{E}_\perp = (-\text{in}, 0, \text{in}) \]

**Proof:** (1) \( \hat{E} \parallel \hat{x} \) lies in the orbital plane.

- \( \hat{E} \perp \hat{x} \)
- Note that the proper rhombus into the parallel polarization

\[ \left( \frac{d^2 \mathcal{P}(t)}{du \, dR} \right)_\parallel = \frac{\omega^2}{4 \pi c^2} \int d\omega \, e^{i \omega \phi} \left[ -x \cdot (x',u) \right] \cdot \left[ -x \cdot (x',u) \right] \]

\[ \left[ -x \cdot (x',u) \right] = \left[ \frac{\partial}{\partial \phi} - x \cdot (x', u) \right] = \hat{x}_\perp \text{Lin} \quad x \perp \hat{x}_\perp \]

\[ \hat{x}(u,t) = e^{\hat{v}_0(u)} e^{i \hat{L}_1 x} \]
\[
\begin{aligned}
\text{Line 1:} & \quad V_2(\tau + \omega_1) V_2(\tau - \omega_1) = \frac{i}{\epsilon} v^2 \left( 2u_0 + 2u_0 \omega_0 \right) \\
\text{Proof:} & \quad \text{If } a(\tau) \text{ is a } \tau \text{-periodic } \text{function, then}
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 2:} & \quad -i\epsilon \left[ \left( V_2(\tau + \omega_1) - V_2(\tau - \omega_1) \right) \right] \left( \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n - \omega_0)^2} \right) e^{-i\omega_0 \tau}
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 3:} & \quad \text{Let } f(\tau) \text{ be } \tau \text{-periodic over some subinterval period}
\end{aligned}
\]

\[
\begin{aligned}
\text{Proof:} & \quad \text{If } a(\tau) \text{ is } \tau \text{-periodic, then}
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 4:} & \quad \text{L.L.} - \frac{\epsilon}{m} e^{-i\omega_0 \tau} \sum_{n=-\infty}^{\infty} \delta_{\omega_n, \omega_0} e^{-i\omega_0 \tau} e^{-i\omega_0 \tau} = e^{-i\omega_0 \tau} e^{-i\omega_0 \tau} = \text{R.L.}
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 5:} & \quad \text{L.L.} = \frac{1}{m} \sum_{n=-\infty}^{\infty} e^{-i(m-n)\omega_0 \tau} e^{-i\omega_0 \tau} = -\frac{1}{2 \epsilon} \left( e^{-i(m-1)\omega_0 \tau} + e^{-i(m-1)\omega_0 \tau} \right)
\end{aligned}
\]

\[
\begin{aligned}
\text{Proof:} & \quad \text{If } a(\tau) \text{ is } \tau \text{-periodic, then}
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 6:} & \quad \text{L.L.} - \frac{\epsilon}{m} (\delta_{\omega_0, \omega_0} + e^{-i\omega_0 \tau}) = -\frac{1}{2 \epsilon} \left( e^{-i(m-1)\omega_0 \tau} + e^{-i(m-1)\omega_0 \tau} \right)
\end{aligned}
\]

\[
\begin{aligned}
\text{Line 7:} & \quad \text{R.L.}
\end{aligned}
\]
\[ \left( \frac{d^1 \psi (r)}{du \, dr} \right)_{||} = \frac{\omega^2 e^2}{4 \pi c^3} \int d^2 \xi \, \frac{1}{\xi} \left[ \psi \frac{D}{c} \frac{1}{2} \left( \left[ \frac{1}{2} \psi_{m+1}^2 + \frac{1}{2} \psi_{m-1}^2 \right] - \psi_{m+1} \psi_{m-1} \right) \right] \delta (\mu - \mu_0) \]

\[ \left( \frac{d \rho_n}{d \mu} \right) \text{ is seen by the experimenter for } R \text{ will} \]

\[ \frac{1}{2} \left( \frac{\partial \psi_{m+1}}{\partial \mu} + \frac{\partial \psi_{m-1}}{\partial \mu} \right) - \frac{\partial}{\partial \mu} \frac{\psi_{m+1}}{\partial \psi_{m-1}} \]

\[ \text{upheld by} \]

\[ \frac{1}{2} \left( \psi_{m+1}^2 + \psi_{m-1}^2 \right) - \psi_{m+1} \psi_{m-1} = \frac{1}{2} \left( \psi_{m+1}^2 - \psi_{m-1}^2 \right) = 2 \left( \psi_{m} \right)^2 \]

\[ \text{The poinsettia into the } m^{th} \text{ harmonic will}\]

\[ \text{potential is given by} \]

\[ \left( \frac{d \rho_n}{d \mu} \right)_{||} = \frac{\omega e^2}{8 \pi} \left( \frac{\psi}{c} \right)^2 \left[ \psi_{m} \left( \frac{\psi}{c} \right) \right]^2 \]

\[ \text{Note: (1) This is the first of the two types of the harmonic for } \frac{d \rho_n}{d \mu} \text{ on p. 96.} \]

\[ \text{Wollera: The poinsettia into the } m^{th} \text{ harmonic will}\]

\[ \text{potential is given by the second} \]

\[ \left( \frac{d \rho_n}{d \mu} \right)_{||} = \frac{\omega e^2}{8 \pi} \left( \frac{\psi}{c} \right)^2 \left( \frac{\psi_{m} \left( \frac{\psi}{c} \right)}{\frac{\psi}{c} + \psi_{m}} \right) \]

\[ \text{Note: (1) This is the first of the two types of the harmonic for } \frac{d \rho_n}{d \mu} \text{ on p. 96.} \]
Plot the two contributions:

\begin{align*}
\text{vc} &= 0.99 \\
\text{gamma} &= 1 / (1 - \text{vc}^2)^{1/2} \\
\text{gamma}^3 \\
m &= \text{Floor}[\text{gamma}^3] \\
J[m\_\_, x\_\_] &= \text{BesselJ}[m, x] \\
J'\text{Prime}[m\_\_, x\_\_] &= (\text{BesselJ}[m - 1, x] - \text{BesselJ}[m + 1, x]) / 2 \\
f\text{par}[\text{theta}\_\_] &= m^2 (J'\text{Prime}[m, m \cdot \text{vc} \cdot \text{Sin}[\text{theta}]]^2 \\
\text{Plot}[f\text{par}[x], \{x, 0, \pi\}, \text{PlotRange} \to \text{All}] \\
f\text{perp}[\text{theta}\_\_] &= m^2 (J[m, m \cdot \text{vc} \cdot \text{Sin}[\text{theta}]] / (\text{vc} \cdot \text{Tan}[\text{theta}])^2 \\
\text{Plot}[f\text{perp}[x], \{x, 0, \pi\}, \text{PlotRange} \to \text{All}] \\
\text{Plot}[f\text{par}[x] + f\text{perp}[x], \{x, 0, \pi\}, \text{PlotRange} \to \text{All}] \\
\end{align*}
Remark: (2) \( (d^2P/d\chi)^2 \) has a maximum at minimum, respecting at \( \chi = \frac{\pi}{2} \). This is a tell-tale sign of stability, indicating that it is important for astrophysical observations.
7.2 Scattering

**Def.** Scattering of electromagnetic radiation by an electron is called Thomson scattering. (J.J. Thomson 1856-1940, Nobel Prize 1906)
Weider-Ampere-Minkowski particle will change e, mess in:

\[ \mathbf{\dot{m}} = -e \mathbf{E} \]

\[ \mathbf{P}_{\text{scat}} = \frac{2e^2}{3c^3} (\mathbf{v} \times \mathbf{E}) = \frac{2e^2}{3c^3} \left( \frac{e}{mc} \right)^2 \mathbf{E} \cdot \mathbf{v} - \frac{e^4}{3mc^2} \mathbf{E}^2 \]

The angular vector in vacua is \((1, 0, 0, 0)\)

\[ \mathbf{P} = c \mathbf{u} \times \mathbf{E} = \frac{c}{m} \left( \mathbf{E} \times \mathbf{v} \right) \mathbf{u} = \frac{c}{m} \mathbf{E} \cdot \mathbf{v} \mathbf{u} = \left| \mathbf{P} \right| = \frac{c}{m} \mathbf{E}^2 \]


The cross section for Thomson scattering is

\[ \sigma = \frac{80 \pi e^4}{3m^2 c^4} \]


\[
\frac{d \mathbf{P}_{\text{scat}}}{d \mathbf{r}} = \frac{e^2}{40 \pi c^2} \left[ \mathbf{E} \times (\mathbf{x} \times \mathbf{E}) \right] = \frac{e^2}{40 \pi m c^2} \mathbf{E} \cdot \left[ \mathbf{E} \times (\mathbf{x} \times \mathbf{E}) \right]
\]

1° con: \( \mathbf{E} \perp \mathbf{x} \Rightarrow \mathbf{E} \cdot \mathbf{v} = 0 \)

\[ \Rightarrow 1 - (\mathbf{x} \cdot \mathbf{E})^2 = 1 \]

2° con: \( \mathbf{E} \perp \mathbf{x} \) not in same plane

\[ |\mathbf{E} \cdot \mathbf{x}| = \mathbf{w} \Rightarrow 1 - (\mathbf{x} \cdot \mathbf{E})^2 = \mathbf{w} \cdot \mathbf{x} \]
Proposition 2: For unpolarized incident radiation, the inclusive scattering cross section per which angle is

\[ \frac{d\sigma}{d\Omega} = \frac{\pi}{2} \frac{1 + \alpha^2 \frac{E}{m^2 c^2}}{1 - (\frac{E}{m^2 c^2})^2} \]

with \( r_0 = \frac{e^2}{m^2 c^2} \)

Proof: Unpolarized radiation \( \rightarrow \) overlap over the two coms.

\[ 1 - (\frac{E}{m^2 c^2})^2 = \frac{1}{2} \left( 1 + \alpha^2 \frac{E}{m^2 c^2} \right) \]

\[ \Rightarrow \frac{d\sigma}{d\Omega} = \frac{d\Omega_{ Suche } / dR}{1^{\text{th}}} = \frac{\frac{4\pi}{9\alpha^2 c^3} \frac{1}{2} (1 + \alpha^2 \frac{E}{m^2 c^2}) \frac{E}{c} \frac{1}{E} \frac{1}{m^2 c^2} \frac{1}{2} (1 + \alpha^2 \frac{E}{m^2 c^2})}{1^{\text{th}}} \]

which:

\[ \int dR \frac{d\sigma}{d\Omega} = r_0^2 \left( 1 + \frac{1}{2} \epsilon \sqrt{1 + 2 \epsilon} \right) = r_0^2 \left( 1 + \frac{1}{2} \right) = \frac{3}{2} r_0^2 \]

Remark: (3) Theoruses scattering is important in astrophysics (other sources, linear polarization of Crab) and plasma physics (for matter, the electron impels a dipole in a plane).
7.3 Scattering by a bound charge

Now consider the scattering of light by a bound charge $e$, modeled as a Lennard-Jones oscillator with mass $m$ and charge $+e$. Let $u$ be the damping constant.

Proposition: The scattering cross section is 

$$
\sigma = \frac{2\pi}{3} \left( \frac{e^2}{\omega^2} \right)^{1/2} \frac{W^4}{(u_0^2 - u^2)^{1/2} + U^2}
$$

Remark: (1) For $W \gg u_0$, the Thomson cross section from §7.1 is as it must.

Proof: The eq. of motion for the charge is 

$$m \ddot{x} + m u_0^2 x + my \dot{y} = eE$$

will $x(t)$ the position of the charge and $E$ taken at the equilibrium position of the Lennard-Jones.

Remark: (2) This is a Lennard-Jones charge by the external force $eE(t)$, and it means the equilibrium position is in fact unpaired to $E$.

Empirical formula for $E(u)$ is 

$$E(u) = -m \omega_0^2 x(u) + m\omega_0 x(u)I_1 y(u)$$

$$\Rightarrow \ddot{x}(u) = \frac{(e/c^2) \dot{E}(u)}{u_0^2 - u^2 + iy(u)}$$

$$\Rightarrow |\dot{x}(u)|^2 = |\dot{x}(u)|^2 = \frac{e^2}{c^2} |\dot{E}(u)|^2 \frac{u_0^4}{(u_0^2 - u^2)^{1/2} + U^2}$$

Now we use the Lorentz formula, giving:

$$P_{scat} = \frac{2e^2}{c^2} |\dot{x}(u)|^2 = \frac{2e^2}{c^2} \frac{e^2}{c^2} |\dot{E}(u)|^2 x(u)$$
\[ e \left| P \right|^2 = \frac{e^2}{4 \pi} \left| E \right|^2 \] (e. 7.2)

\[ \tau = \frac{\text{phase shift}}{1^2} = \frac{2e^2}{3c^2m^2} \frac{\pi}{c} X(u) = \frac{8\pi}{3} \left( \frac{\varepsilon}{mc^2} \right)^2 X(u) \]

Diagram:

1. \( X(u) \) shows a pronounced \( u=0 \) will violate \( \tau \).

2. For \( u \gg u_0 \), the first oscillation of the wave leaves the beam so no time to respond \( \Rightarrow \tau \sim \frac{1}{u_0} \).

3. For \( u \ll u_0 \), \( \tau \propto u^4 \) (Rayleigh scattering). This is why the sky is blue in clear weather.

4. We have another kind of fixed horizon, when it is dark, due to the rest of the earth itself, see §4.5. This is the problem of "radiation damping" (see Jackson eq. 17 and Schwinger eq. 45.4).