

## Problem Assignment # 1

10/02/2019  
due 10/09/2019**1. Russell's Paradox (B. Russell, 1901)**

a) Consider the set  $M$  defined as the set of all sets that do not contain themselves as an element:  $M = \{x; x \notin x\}$ . Discuss why this is a problematic definition.

b) A less abstract version of Russell's paradox is known as the barber's paradox: Consider a town where all men either shave themselves, or let the barber shave them and don't shave themselves. Now consider the statement

*The barber is a man in town who shaves all men who do not shave themselves, and only those.*

Discuss why this definition of the barber is problematic (assuming there actually is a barber in town).

*hint:* Ask "Does the barber shave himself?"

c) Suppose the definition of the barber is modified to read

*The barber shaves all men in town who do not shave themselves, and only those.*

Discuss what this modification does to the paradox.

(3 points)

**2. Distributive property of the union and intersection relations**

Convince yourself graphically of the distributive property of the relations  $\cup$  and  $\cap$ , ch.1 §1.1 remark (9)(iii). That is, for any three sets  $A, B, C$ ,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(2 points)

**3. Mappings**

Are the following  $f : X \rightarrow Y$  true mappings? If so, are they surjective, or injective, or both?

a)  $X = Y = \mathbb{Z}$ ,  $f(m) = m^2 + 1$ .

b)  $X = Y = \mathbb{N}$ ,  $f(n) = n + 1$ .

c)  $X = \mathbb{Z}$ ,  $Y = \mathbb{R}$ ,  $f(x) = \log x$ .

d)  $X = Y = \mathbb{R}$ ,  $f(x) = e^x$ .

(2 points)

**4. Parabolic Mapping**

Consider  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $f(n) = an^2 + bn + c$ , with  $a, b, c \in \mathbb{Z}$ .

a) For which triplets  $(a, b, c)$  is  $f$  surjective?

b) For which  $(a, b, c)$  is  $f$  injective?

(4 points)