

Problem Assignments # 7

11/13/2019
due 11/20/2019**25. Transformations of tensor fields**

- a) Consider a covariant rank- n tensor field $t_{i_1 \dots i_n}(x)$ and find its transformation law under normal coordinate transformations that is analogous to §5.1 def.1; i.e., find how $\tilde{t}_{i_1 \dots i_n}(\tilde{x})$ is related to $t_{i_1 \dots i_n}(x)$.
- b) Convince yourself that your result is consistent with the transformation properties of (i) a covector x_i (the case $n = 1$), and (ii) the covariant components of the metric tensor g_{ij} .

(4 points)

26. Curl and divergence

Show that the curl and the divergence of a vector field transform as a pseudovector field and a scalar field, respectively.

(3 points)

27. Tensor products, and tensor traces

Prove Propositions 1 and 2 from ch. 1 §5.3.

(4 points)

28. Lindhard function

Consider the function $f : \mathbb{C} \rightarrow \mathbb{C}$ (which plays an important role in the theory of many-electron systems) defined by

$$f(z) = \log \left(\frac{z-1}{z+1} \right)$$

The *spectrum* $f'' : \mathbb{R} \rightarrow \mathbb{R}$ and the *reactive part* $f' : \mathbb{R} \rightarrow \mathbb{R}$ of f are defined by

$$f''(\omega) := \frac{1}{2i} [f(\omega + i0) - f(\omega - i0)] \quad , \quad f'(\omega) := \frac{1}{2} [f(\omega + i0) + f(\omega - i0)]$$

- a) Show that f' and f'' are indeed real-valued functions.
- b) Determine f'' and f' explicitly, and plot them for $-3 < \omega < 3$.
- c) Show that

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{f''(\omega)}{\omega - z} = f(z)$$

(5 points)