Problem Assignments # 8 11/20/2019
due 11/27/2019

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29. **Another causal function**

The function considered in Problem 25. is an example of a class of complex functions called *causal functions* that are important for the theory of many-particle systems. Another member of this class is

$$g(z) = \sqrt{z^2 - 1} - z$$

Determine the spectrum and the reactive part of $g(z)$, and plot them for $-3 < \omega < 3$.

(3 points)

30. **Exponentials**

Consider the exponential function

$$f(z) = e^z = e^{z' + iz''}$$

a) Show that $f(z)$ is analytic everywhere in $\mathbb{C}$.

b) Convince your self explicitly that the real and imaginary parts of f obey Laplace's differential equation.

c) Show that $df/dz|_z = f(z)$.

d) Show that $\cos z$ and $\sin z$, defined by

$$\cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right) \quad , \quad \sin z = \frac{1}{2i}\left(e^{iz} - e^{-iz}\right)$$

are analytic everywhere in $\mathbb{C}$, and that

$$\frac{d}{dz}\cos z = -\sin z \quad , \quad \frac{d}{dz}\sin z = \cos z \; .$$

(4 points)

32. **1-d Fourier transforms**

Consider a function f of one real variable x. Calculate the Fourier transforms of the following functions:

a) $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{otherwise} \end{cases} \quad .$

b) $f(x) = \begin{cases} 1 - |x| & \text{for } |x| \le 1 \\ 0 & \text{otherwise} \end{cases} \quad .$

c) $f(x) = e^{-(x/x_0)^2} \quad .$

(3 points)

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33. **3-d Fourier transforms**

Consider a function f of one vector variable $\boldsymbol{x} \in I\!R^3$. The Fourier transform $\hat{f}$ of f is defined as

$$\hat{f}(\boldsymbol{k}) = \int d\boldsymbol{x}\; e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, f(\boldsymbol{x}) \quad .$$

Calculate the Fourier transforms of the following functions:

a)

$$f(\boldsymbol{x}) = \begin{cases} 1 & \text{for } r < r_0 \\ 0 & \text{otherwise} \end{cases} \quad . \qquad (r = |\boldsymbol{x}|)$$

b)

$$f(\boldsymbol{x}) = 1/r \quad .$$

hint: Consider $g(\boldsymbol{x}) = \frac{1}{r}\, e^{-r/r_0}$ and let $r_0 \to \infty$.

(3 points)