36. Potential of a uniformly moving charge
Consider a charge $e$ moving uniformly along the $x$-axis with velocity $v$: $X(t) = (vt, 0, 0)$. Determine the Liénard-Wiechert potentials explicitly, and show that the result is the same as the one obtained in ch. 2 §2.4 by means of a Lorentz transformation.

(6 points)

37. Potentials in Coulomb gauge
Consider the potentials $\phi$ and $A$ in the Coulomb gauge, i.e., the field equations from ch.4 §1.2 proposition 2. Show explicitly that the resulting asymptotic electric and magnetic fields are the same as those calculated in the Lorenz gauge in ch.4 §3.

hint: Show that the scalar potential does not contribute to the electric field, and show that the asymptotic vector potential now reads

$$A(x, t) = -\hat{x} \times \left[ \hat{x} \times \frac{1}{rc} \int dy j(y, t) \right]$$

instead of the expression derived in ch. 4 §3.1. Then calculate the fields.

(8 points)

38. Radiation from cyclotron motion
Consider a point mass $m$ with charge $e$ that moves in a plane perpendicular to a homogeneous magnetic field $B$. Assume nonrelativistic motion, $v \ll c$

a) Find the power radiated by the particle.

b) Show that the energy of the particle decreases with time according to $E(t) = E_0 e^{-t/\tau}$, and determine the timescale $\tau$.

c) Find $\tau$ in seconds for an electron in a magnetic field of 1 Tesla.

(4 points)