9. Particle in homogeneous $E$ and $B$ fields

Consider a point particle (mass $m$, charge $e$) in homogeneous fields $\mathbf{B} = (0, 0, B)$ and $\mathbf{E} = (0, E_y, E_z)$. Treat the motion of the particle nonrelativistically.

a) Show that the motion in $z$-direction decouples from the motion in the $x$-$y$ plane, and find $z(t)$.

b) Consider $\xi := x + iy$. Find the equation of motion for $\xi$, and its most general solution.

*hint:* Define the *cyclotron frequency* $\omega = eB/mc$, and remember how to solve inhomogeneous ODEs.

c) Show that the time-averaged velocity perpendicular to the plane defined by $\mathbf{B}$ and $\mathbf{E}$ is given by the *drift velocity*

$$\langle v \rangle = e\mathbf{E} \times \mathbf{B}/B^2$$

Show that $E_y/B \ll 1$ is necessary and sufficient for the non relativistic approximation to be valid.

d) Show that the path projected onto the $x$-$y$ plane can have three qualitatively different shapes, and plot a representative example for each.

(6 points)

10. Harmonic oscillator coupled to a magnetic field

Consider a charged 3-d classical harmonic oscillator (oscillator frequency $\omega_0$, charge $e$). Put the oscillator in a homogeneous time-independent magnetic field $\mathbf{B} = (0, 0, B)$. Show that the motion remains oscillatory, and find the oscillation frequencies in the directions parallel and perpendicular, respectively, to $\mathbf{B}$.

(4 points)

11. Relativistic motion in parallel electric and magnetic fields

Consider a relativistic charged particle (mass $m$, charge $e$) in parallel homogeneous electric and magnetic fields $\mathbf{E} = (0, 0, E)$, $\mathbf{B} = (0, 0, B)$.

a) Show that the equation of motion for the $z$-component of the momentum $p_z$ decouples from $p_x$ and $p_y$, and that the momentum perpendicular to the $z$-axis is a constant of motion: $p_x^2 + p_y^2 = p_{\perp}^2 = \text{const}$.

b) Choose the zero of time such that $p_z(t = 0) = 0$, and show that with a suitable chosen origin the $z$-component of the particle’s position can be written

$$z(t) = \frac{1}{cE} \sqrt{T_0^2 + c^2e^2E^2t^2}$$

where $T_0$ is the kinetic energy (i.e., the energy of the particle without the potential energy due to the fields) at time $t = 0$.

*hint:* If you have trouble, recall Einstein’s law of falling bodies from PHYS 611. You can find my version at http://pages.uoregon.edu/dbelitz/teaching/2013_14/PHYS_611-4/ , Assignment # 5, Problem 21.

.../over
c) Introduce a parameter $\varphi$ via $d\varphi/dt = ceB/T(t)$, with $T(t)$ the time-dependent kinetic energy. Show that the orbit of the particle can be represented in the parametric form

$$x = \frac{c p_{\perp}}{eB} \sin \varphi, \quad y = \frac{c p_{\perp}}{eB} \cos \varphi, \quad z = \frac{T_0}{eE} \cosh(E\varphi/B)$$

and explicitly find the relation between $\varphi$ and $t$.

*hint:* Consider $\pi := p_x + ip_y$ and note that $|\pi| = p_{\perp} = \text{const.}$ by the result of part a).

d) Describe and visualize the orbit, and discuss the motion in the limits of large and small times.

(14 points)