Remark: (3) \( \chi_{\text{die}} = - \frac{1}{3} \chi_{\text{pore}} \Rightarrow \chi = \chi_{\text{die}} + \chi_{\text{pore}} = \frac{2}{3} \chi_{\text{pore}} > 0 \)

(4) Extend A-field has two effects:
(a) it has to align the spins \( \Rightarrow \) pore magnetism
(b) it induces a diamagnetic (here's why!) moment due to
the orbital motion of the electrons \( \Rightarrow \) diamagnetism

(5) For an ideal doped \( \langle \phi \rangle = 0 \), \( \mathcal{D} \mid 0 \), the specific heat \( C_V \) is
\( X \) of heat \( X_{\text{pore}} \); \( X_{\text{die}} \); and the density of states on the
Fermi level:

\[
N(V) = \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} N_0 \frac{\varepsilon}{e^{\frac{\varepsilon}{T}} - 1} = \frac{V}{2} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[
\frac{\partial N}{\partial V} = \frac{\varepsilon}{2\pi} N_0 \frac{\varepsilon}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow C_V / V = \frac{\frac{V}{2} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}}{\frac{V}{2} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}} = \frac{\frac{T}{2}}{\lambda} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow X_{\text{pore}} = - \frac{2}{3} X_{\text{die}} = - \frac{2}{3} \lambda N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow X_{\text{pore}} = - \frac{2}{3} \lambda N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow C_V / V = \frac{\frac{V}{2} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}}{\frac{V}{2} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}} = \frac{\frac{T}{2}}{\lambda} N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow X_{\text{pore}} = - \frac{2}{3} X_{\text{die}} = - \frac{2}{3} \lambda N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]

\[\Rightarrow X_{\text{pore}} = - \frac{2}{3} \lambda N_0 \frac{T}{e^{\frac{T}{\lambda}} - 1}
\]
Remark: (1) We have put $v_0 = 0$ a.k.a. (otherwise, just redefine $\mu$)

(2) Exercise (for $\epsilon$) $\rightarrow \epsilon_i \rightarrow e^{-\epsilon_i}$, then apply the same for $\epsilon_i > 0$ or $\epsilon_i < 0$.

(3) $N_0 (\mu \rightarrow 0) \rightarrow \infty$, ad proba $\epsilon \rightarrow 0 \leftarrow \mu (\mu \rightarrow 0) \rightarrow 0$ will $\mu_0 \rightarrow 0 \rightarrow \infty$ clearly. $N_0$ may be divergent!

\[
N^1 = \frac{gV}{\sqrt{2\pi}} \left( \frac{m_a^{-2}}{2\pi t^2} \right)^{3/4} \frac{1}{2} \int_0^\infty dx \frac{x^{-1/2}}{e^{-x} - 1} = \frac{gV}{\sqrt{2\pi}} \left( \frac{m_a^{-2}}{2\pi t^2} \right)^{3/4} \frac{1}{2} \left( -4 \right)
\]

where $3(x) \equiv \frac{1}{2} \int_0^\infty dx \frac{x^{-1/2}}{e^{-x} - 1}$ (for $x > 0$)

Theorem 1:

\[
\text{discriminant of } 3(x): 3(0) = \frac{1}{2} \int_0^\infty dx \frac{x^{-1/2}}{e^{-x} - 1} = \frac{1}{2} \frac{\Gamma(1/2)}{\Gamma(2)} \frac{\Gamma(3/2)}{\Gamma(3)} = 2.112.
\]

\[e^x > 1 \Rightarrow x > 0 \rightarrow 3(x) \text{ decays monotonically with } x \]

\[3(x \rightarrow 0) \rightarrow \frac{1}{2} e^{-x} \int_0^\infty dx \frac{x^{-1/2}}{e^{-x} - 1} = e^{-x}
\]

\[
N^1 \leq N_{\text{max}} = gV \left( \frac{m_a^{-2}}{2\pi t^2} \right)^{3/4} \frac{1}{2} \left( -4 \right) = gV \left( \frac{m_a^{-2}}{2\pi t^2} \right)^{3/4} s(1/2)
\]

Defining $T_0$ by

\[
gV \left( \frac{m_a^{-2}}{2\pi t^2} \right)^{3/4} s(1/2) = N
\]

\[
r_{a_0} = \frac{2\pi t^2}{m} (N)^{1/3} \left( \frac{a}{s(1/2)} \right) \frac{k t}{\sqrt{2}} \frac{\hbar}{m^{3/2}} \frac{\hbar^{3/2}}{2\pi m^{1/2}} = \frac{2\pi}{s(1/2)^{1/3}},
\]

For $T < T_0$, the excited states cannot overcome all of the potential.

Remark: (4) Cf. problem 17.2.
Proposition: If $A$ is a ideal then yes if $T < T_0$, we have

$$N_{T_0} = \frac{\frac{a}{q_0}}{\nu} \nu^{1/2}$$

$$a = \frac{\nu}{\sqrt{1 - \nu^2}} \nu^{1/2}$$

$$\nu = \frac{N}{V}$$

a final prothesis \( N_0 \backslash N \) of all particles in the grand
stark, we for $N \to \infty$.

Remark: (5) Problem $\partial T \to T_0$ in the hypothesis when $\mu \to 0$.

(6) The solution of a final density of particles in the grand
stark is called Bose-Einstein condensation.

1.2 Unmixed particles at particle number, for $T \leq T_0$

$\S 2.1 \Rightarrow N_0 = N - N' = \frac{1}{e - T_0 - 1} \Rightarrow e^{-T_0} = 1 + \frac{1}{N - N'}$

$$\mu = A_0 \tau \ln \left( 1 + \frac{1}{N - N'} \right) = A_0 \tau \ln \left( 1 + \frac{1}{N - N'_{\text{nev}}} \right)$$

Remark: (1) For finite $N$, $\mu (T < T_0)$ remains a small negative value
so that the result under $N_0 = N - N'_{\text{nev}}$ of particles is
unmodified in the grand stark.

(2) In the thermodynamic limit, $N \to \infty$ while $N'_{\text{nev}} \to \infty$

$$N - N'_{\text{nev}} = \infty \Rightarrow \mu (T < T_0) = 0$$

Consider the $T$-dependence of $N'_{\text{nev}}$ at $N_0$.

$$N'_{\text{nev}} = \frac{3}{2} \ln \left( \frac{\frac{1}{\nu} (\frac{\nu}{\sqrt{1 - \nu^2}}) \nu^{1/2} + \nu} {3^{1/2}} \right)$$

$$\frac{3}{2} \ln \left( \frac{N_0 \nu^2}{(\frac{\nu}{\sqrt{1 - \nu^2}}) \nu^{1/2} + \nu} {3^{1/2}} \right)$$

$$\frac{3}{2} \ln \left( \frac{N_0 \nu^2}{(\frac{\nu}{\sqrt{1 - \nu^2}}) \nu^{1/2} + \nu} {3^{1/2}} \right)$$
\[
\frac{N'}{N} = \left(\frac{T}{T_0}\right)^{3/2}, \quad \frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^{3/2}
\]

Work: (2) \( n, m \) for He-4

\( T_0 = 2.12 \, K \)

(4) He-4 has ideal gas properties below \( T_0 = 2.12 \, K \)

(5) Point 1: \( \text{DEC} \) has been rounded in very weakly, strongly

### 3.2 The specific heat of an ideal gas

\( T < T_0 \):

\[
U = \sum E_h \, n_h \cdot \left( E_h \right) = \sum \frac{\sqrt{m \cdot E_{3/2}}}{E^{1/2}} \int_0^E \frac{E_{3/2}}{E^{1/2}} \, dE \left( \frac{E_{3/2}}{E^{1/2}} \right)
\]

\[
\frac{dU}{dT} = \frac{\sqrt{m} \cdot x_{3/2}}{x_{1/2}} \frac{1}{x^{1/2}} \int_0^x \frac{x_{3/2}}{x^{1/2}} \, dx \left( \frac{x_{3/2}}{x^{1/2}} \right)
\]

\[
\frac{M}{S(3/2)} = \left( \frac{T}{T_0} \right)^{3/2} \, \frac{x_{3/2}}{x_{1/2}} \frac{1}{x^{1/2}} \frac{S(5/2)}{S(3/2)}
\]

\[
N_A \, T \, (T/T_0)^{3/2} \, \frac{1}{2} \, \frac{S(5/2)}{S(3/2)} = 0.77... \times N_A \, T \, (T/T_0)^{3/2}
\]

\[
C_V = \frac{\partial U}{\partial T} = N_A \, T \, (T/T_0)^{3/2} \, \frac{1}{2} \, \frac{S(5/2)}{S(3/2)} \quad (T < T_0)
\]

\( T > T_0 \): Now we make \( f(T) \)

\[
f_{\text{H}} \left( T \to T_0^+ \right) = - \left( \frac{\Delta S(3/2)}{n_0} \right) \, \frac{1}{2} \, \frac{m}{u_s} \, \left[ \left( \frac{T}{T_0} \right)^{3/2} - 1 \right]^2
\]
\[ \frac{\partial v}{\partial t} = 0 \quad \frac{\partial P}{\partial t} = 0 \]

\[ u = \frac{1}{2} \left( v + \frac{\partial P}{\partial x} \right) \]

\[ \frac{\partial u}{\partial t} = \frac{1}{2} \left( \frac{\partial^2 P}{\partial x^2} - \frac{\partial^2 P}{\partial y^2} \right) \]

\[ \frac{\partial^2 P}{\partial x^2} = 0 \quad \frac{\partial^2 P}{\partial y^2} = 0 \]

\[ N_0 = \frac{1}{A} \int \left( \frac{\partial P}{\partial x} \right) dx \]

\[ N_1 = \int \left( \frac{\partial P}{\partial x} \right) dx \]

\[ N_1 = \frac{1}{A} \int \left( \frac{\partial P}{\partial x} \right) dx \]

\[ \text{Proof:} \]

\[ N = \frac{1}{N_0} \int \frac{\partial P}{\partial x} \]
work: (2) We know \( \frac{\partial u}{\partial T} \) from the calculation for \( \bar{T} \rightarrow T \):
\[
\frac{\partial u}{\partial T} = N A_T \left( \frac{m}{k_B} \right)^{3/2} S^2 \left( \frac{m}{k_B} \right) \left( \frac{m}{k_B} \right)^{1/2} \lambda \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right] \frac{m}{k_B} \frac{T}{T_0}^{3/2}
\]
for \( T < T_0 \) this is \( C_v \), for \( T > T_0 \) it is just the contribution to \( C_v \).
\[
C_v (T \rightarrow T_0) = \frac{\partial u}{\partial T} = \frac{\partial u}{\partial T_0} \left( T \rightarrow T_0 \right) + \frac{9}{4} \lambda \left( \frac{m}{k_B} \right)^{1/2} N A_T \left( \frac{m}{k_B} \right)^{3/2} \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right]
\]
\[
= N A_T \left[ \frac{15}{4} \left( \frac{m}{k_B} \right)^{3/2} \right] \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right] - \frac{9}{8} \lambda \left( \frac{m}{k_B} \right)^{1/2} N A_T \left( \frac{m}{k_B} \right)^{3/2} \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right]
\]
\[
C_v (T_0) = N A_T \left[ \frac{15}{4} \left( \frac{m}{k_B} \right)^{3/2} \right] \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right] - \frac{9}{8} \lambda \left( \frac{m}{k_B} \right)^{1/2} N A_T \left( \frac{m}{k_B} \right)^{3/2} \left[ 1 - \left( \frac{T}{T_0} \right)^{3/2} \right]
\]

work: (3) \( C_v (T) \) is continuous, but not differentiable at \( T = T_0 \).

(4) In He, \( C_v (T) \) diverges at \( T \rightarrow T_0 \) (about very slowly),

the "blackbody point". The correct law is a strange inequality.

4.1 Planck's law

Consider photons in equilibrium with matter. Planck's law \( \nu \rightarrow E_B \) statistics applies.

work: (1) Photon-photon interaction is extremely weak (mediated only by virtual \( e^-e^+ \) pairs) as soon as the temperature is near the Planckian equilibrium.

(2) Phason gas can be understood either if the number of, e.g., a

dish gas, or the walls of a cavity when any role is to absorb...