1. Phase flow
Consider a point mass in $d = 1$ subject to a constant force $f$, so that the equation of motion is
\[ \ddot{q}(t) = \frac{f}{m} . \]
a) Determine the phase flow. Does Liouville’s theorem hold?
b) Does Poicaré’s theorem hold? Justify your answer.

2. Circle translations
Let $S_1$ be the 1-sphere (a.k.a. circle), and let $g_\alpha$ be a rotation of $S_1$ about its center through an angle $\alpha$. Show that
a) There exists an $n \in \mathbb{Z}$ so that any point $x \in S_1$ returns to its original position after $n$ rotations if and only if $\alpha/2\pi \in \mathbb{Q}$.
b) If $\alpha/2\pi \notin \mathbb{Q}$, then the set of points $\{g^kx; k \in \mathbb{Z}\}$ is dense in $S_1$.

3. Torus translations
Let $T_2 = S_1 \times S_1$ be the 2-torus (i.e., the ordinary doughnut, or the set of points $x = (\phi_1, \phi_2), \phi_i \in \mathbb{R}$ ($i = 1, 2$) and $(\phi_1, \phi_2) = (\psi_1, \psi_2)$ if and only if $(\phi_i - \psi_i)/2\pi \in \mathbb{Z}$ ($i = 1, 2$).) Let torus translations be defined by
\[ f_t(\phi_1, \phi_2) = (\phi_1 + c_1 t, \phi_2 + c_2 t), \quad t \in \mathbb{R}, \quad 0 \neq c_i \in \mathbb{R}, \quad (i = 1, 2) . \]
The winding line $\ell$ through a point $x_0 \in T_2$ is defined as the set
\[ \ell = \{ x \in T_2; x = f_t(x_0), \ t \in \mathbb{R} \} . \]
Show that
a) $\ell$ is closed if and only if $c_1/c_2 \in \mathbb{Q}$.
b) If $\ell$ is not closed, then $\ell$ is dense in $T_2$.

4. Powers of 2
Consider the first digits of the numbers $2^n$ ($n = 0, 1, 2, \ldots$): $1, 2, 4, 8, 1, 3, 6 \ldots$
a) Does the number 7 appear in this series? If so, how often?
b) Let $N_7(n)$ and $N_8(n)$ be the number of times 7 and 8, respectively, appear in the first $n$ numbers. Find the relative frequency of occurrence, $\lim_{n \to \infty} N_7(n)/N_8(n)$.