PHYS 614 Statistical Mechanics S 2022

Problem Assignment # 1 03/31/2022
due 04/07/2022

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notation: $\mathcal{Z}$ = integers, $\mathcal{Q}$ = rationals, $\mathcal{R}$ = reals.

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1. Phase flow

Consider a point mass in $d = 1$ subject to a constant force f, so that the equation of motion is

$$\ddot{q}(t) = f/m \quad .$$

a) Determine the phase flow. Does Liouville's theorem hold?

b) Does Poicaré's theorem hold? Justify your answer.

(4 points)

2. Circle tranlations

Let S_1 be the 1-sphere (a.k.a. circle), and let g_α be a rotation of S_1 about its center through an angle α. Show that

a) There exists an $n \in \mathcal{Z}$ so that any point $x \in S_1$ returns to its original position after n rotations if and only if $\alpha/2\pi \in \mathcal{Q}$.

b) If $\alpha/2\pi \notin \mathcal{Q}$, then the set of points $\{g^k x;\ k \in \mathcal{Z}\}$ is dense in S_1.

(5 points)

3. Torus translations

Let $T_2 = S_1 \times S_1$ be the 2-torus (i.e., the ordinary doughnut, or the set of points $x = (\phi_1, \phi_2)$, $\phi_i \in \mathcal{R}$ $(i = 1, 2)$, and $(\phi_1, \phi_2) = (\psi_1, \psi_2)$ if and only if $(\phi_i - \psi_i)/2\pi \in \mathcal{Z}$ $(i = 1, 2)$.) Let *torus translations* be defined by

$$f_t(\phi_1, \phi_2) = (\phi_1 + c_1 t, \phi_2 + c_2 t), \quad t \in \mathcal{R}, \quad 0 \neq c_i \in \mathcal{R}, \quad (i = 1, 2) \quad .$$

The *winding line* ℓ through a point $x_0 \in T_2$ is defined as the set

$$\ell = \{x \in T_2;\ x = f_t(x_0),\ t \in \mathcal{R}\} \quad .$$

Show that

a) ℓ is closed if and only if $c_1/c_2 \in \mathcal{Q}$.

b) If ℓ is not closed, then ℓ is dense in T_2.

(4 points)

4. Powers of 2

Consider the first digits of the numbers 2^n $(n = 0, 1, 2, \ldots)$: $1, 2, 4, 8, 1, 3, 6 \ldots$

a) Does the number 7 appear in this series? If so, how often?

b) Let $N_7(n)$ and $N_8(n)$ be the number of times 7 and 8, respectively, appear in the first n numbers. Find the relative frequency of occurrence, $\lim_{n\to\infty} N_7(n)/N_8(n)$.

(6 points)