PHYS 610 Mathematical Methods for Scientists F 2024

Final Exam 12/09/2024 - 12/12/2024

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This is a limited-time open-book exam. You can use any inanimate resources you like, but please don't get help from live ones. (This includes a prohibition of "proofreading" by third parties.)

You can choose any contiguous twentyfour-hour period you like for working on the exam, but once you open it you must finish it within the next twentyfour hours. When you are done, scan it and email it to dbelitz@you-know-where. Please make sure the document is no larger than 5MB. The exam is due no later than Thursday, Dec. 12, 5pm PST. Since different people will be working on the exam at different times I will not answer any questions, neither privately nor publicly, once the exam has been published. If you feel there is something wrong or ambiguous about the formulation of a problem, clearly state your assumptions and keep going.

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1. **Complex analysis** (16 pts)

*Note:* If, in any part of this problem, parts of the contour you wish to integrate over obviously do not contribute, say so and give a simple argument for why that's true (e.g., quote a relevant statement in the notes). You don't have to prove that it's true from scratch again. Similarly, if you feel that the analytic structure of a function is obvious, you can just state what it is without proof.

*Credit Breakdown:* 3 pts each for a), b), c); 7 pts for d)

a) Consider the complex function

$$f(z) = \frac{1}{(1+z^2)^3}$$

Briefly discuss the analytic structure of $f$, classify the singularities, and find the residues in the poles.

b) Explicitly construct the Laurent series for $f$ up to and including the constant term in the vicinity of a pole of your choice and verify the value of the residue you found in part a).

c) Use the residue theorem to evaluate the integral

$$I = \int_0^\infty dx\, f(x)$$

d) Consider the integral

$$J = \int_{-\infty}^\infty dx\, \frac{\cos x}{1-x^2}$$

Interpret this integral in a Cauchy principal value sense in analogy to Problem II.3.2 e) and evaluate it using complex analysis. If you believe that parts of your contours do not contribute, see the note above.

**2. Minkowski tensors** (6 pts: 2 pts each for parts a), b), c) )

Let $F$ be an antisymmetric rank-2 tensor in the Minkowski space $M_4$ with metric $g = (+,-,-,-)$.

a) Write $F$ in terms of its covariant components $F_{\mu\nu}$. How many independent components are there?

b) Express the contravariant components $F^{\mu\nu}$, and the mixed components $F^\mu{}_\nu$ and $F_\mu{}^\nu$, in terms of the covariant ones.

c) What are the transformation properties of the objects $I = F^{\mu\nu}F_{\mu\nu}$ and $J = \epsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda}$ under normal coordinate transformations? (Here $\epsilon^{\mu\nu\kappa\lambda}$ is the 4-dimensional Levi-Civita symbol.) You can just state the answer, you don't need to prove that what you say is true.