PHYS 610 Mathematical Methods for Scientists F 2024

Problem Assignment # 3 10/16/2024
due 10/23/2024

I.2.3 The group S_3 (7 pts)

a) Compile the group table for the symmetric group S_3. Is S_3 abelian?

b) Find all subgroups of S_3. Which of these are abelian?

I.2.4 Subgroups (5 pts)

Let $(G, \vee)$ be a group and let $H \subset G$ with $H \neq \emptyset$. Show that H is a subgroup of G if and only if $a, b \in H$ implies $a \vee b^{-1} \in H$.

I.3.1. Fields

a) Show that the set of rational numbers $\mathbb{Q}$ forms a commutative field under the ordinary addition and multiplication of numbers.

b) Consider a set F with two elements, $F = \{\theta, e\}$. On F, define an operation "plus" $(+)$, about which we assume nothing but the defining properties

$$\theta + \theta = \theta \quad , \quad \theta + e = e + \theta = e \quad , \quad e + e = \theta$$

Further, define a second operation "times" $(\cdot)$, about which we assume nothing but the defining properties

$$\theta \cdot \theta = e \cdot \theta = \theta \cdot e = \theta \quad , \quad e \cdot e = e$$

Show that with these definitions (and **no** additional assumptions), F is a field.

I.4.1 Function space

Consider the set C of continuous functions $f : [0, 1] \to \mathbb{R}$. Show that by suitably defining an addition on C, and a multiplication with real numbers, one can make C an additive vector space over $\mathbb{R}$.

(2 points)