PHYS 622 Classical Electrodynamics W 2025

Problem Assignment # 4 01/29/2025
due 02/05/2025

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1.2.1. **Energy-momentum tensor**

Consider the electromagnetic field in the absence of matter.

a) Show that the tensor field

$$H_\mu^{\ \nu}(x) = (\partial_\mu A_\alpha(x))\, \frac{\partial \mathcal{L}}{\partial\,(\partial_\nu A_\alpha(x))} - \delta_\mu^{\ \nu}\, \mathcal{L}$$

obeys the continuity equation

$$\partial_\nu\, H_\mu^{\ \nu}(x) = 0 \quad (*)$$

*note:* Notice that $H_\mu^{\ \nu}(x)$ is a generalization of Jacobi's integral in Classical Mechanics.

b) Show that (*) also holds for

$$\tilde{T}_\mu^{\ \nu} = H_\mu^{\ \nu} + \partial_\alpha \psi_\mu^{\ \nu\alpha}$$

where $\psi_\mu^{\ \nu\alpha}$ is any tensor field that is antisymmetric in the second and third indices, $\psi_\mu^{\ \nu\alpha}(x) = -\psi_\mu^{\ \alpha\nu}(x)$.

c) Show that $\psi_\mu^{\ \nu\alpha}$ can be chosen such that $\tilde{T}_\mu^{\ \nu}(x) = T_\mu^{\ \nu}(x)$, which provides an alternative proof that $T_\mu^{\ \nu}(x)$ obeys (*).

(5 points)

1.2.2. **Energy-momentum conservation in the presence of matter**

Prove the corollary of ch. 1 §2.3: In the presence of matter, the energy-momentum tensor obeys the continuity equation

$$\partial_\nu T_\mu^{\ \nu}(x) = \frac{-1}{c}\, F_\mu^{\ \nu}(x)\, J_\nu(x)$$

(2 points)

1.2.3. **Energy-momentum tensor for a massive scalar field**

Consider the massive scalar field from ch. 0 §2.5:

$$\mathcal{L} = \frac{1}{2}\,(\partial_\mu \varphi)\,(\partial^\mu \varphi) - \frac{m^2}{2}\,\varphi^2$$

and the tensor field $H_\mu^{\ \nu}$ defined analogously to Problem 1.2.1:

$$H_\mu^{\ \nu} = (\partial_\mu \varphi)\, \frac{\partial \mathcal{L}}{\partial\,(\partial_\nu \varphi)} - \delta_\mu^{\ \nu}\, \mathcal{L}$$

Determine $H_\mu^{\ \nu}$ explicitly and show that

$$\partial_\nu H_\mu^{\ \nu} = 0$$

*hint:* Use the Euler-Lagrange equation determined in ch. 0 §2.5.

(2 points)

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1.2.4. **Coulomb gauge**

Consider the 4-vector potential $A^\mu(x) = (\varphi(x), \boldsymbol{A}(x))$. Show that one can always find a gauge transformation such that

$$\boldsymbol{\nabla} \cdot \boldsymbol{A}(x) = 0$$

This choice is called *Coulomb gauge*.

(2 points)