I.1.1 Russell's Paradox (B. Russell, 1901) (3 pts)

a) Consider the set M defined as the set of all sets that do not contain themselves as an element: $M = \{x; x \notin x\}$. Discuss why this is a problematic definition.

b) A less abstract version of Russell's paradox is known as the barber's paradox: Consider a town where all men either shave themselves, or let the barber shave them and don't shave themselves. Now consider the statement

The barber is a man in town who shaves all men who do not shave themselves, and only those.

Discuss why this definition of the barber is problematic (assuming there actually is a barber in town).

hint: Ask "Does the barber shave himself?"

c) Suppose the definition of the barber is modified to read

The barber shaves all men in town who do not shave themselves, and only those.

Discuss what this modification does to the paradox.

Solution

a) Suppose M contains itself as an element. Then, by its definition, it does not.
Suppose M does not contain itself as an element. Then, again by its definition, it does.
There is no third possibility, so the definition is logically self-contradictory. 1pt

b) Suppose the barber shaves himself. This means that he is shaved by the barber and hence does **not** shave himself.
Suppose the barber does not shave himself. Then he is shaved by the barber and hence **does** shave himself.
This is the same logical problem as in part a). 1pt

c) By dropping the requirement that the barber is "a man in town" the problem goes away: The barber may be a woman, or a man from a different town. 1pt
Note: Another logically possible conclusion is that the town has no barber.