I.1.2 Distributive property of the union and intersection relations (5 pts)

a) Prove the distributive properties from ch.I §1.1 Remark 5 viz.,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

for any three sets A, B, C.

b) Illustrate these properties by means of representative Venn diagrams.

Solution

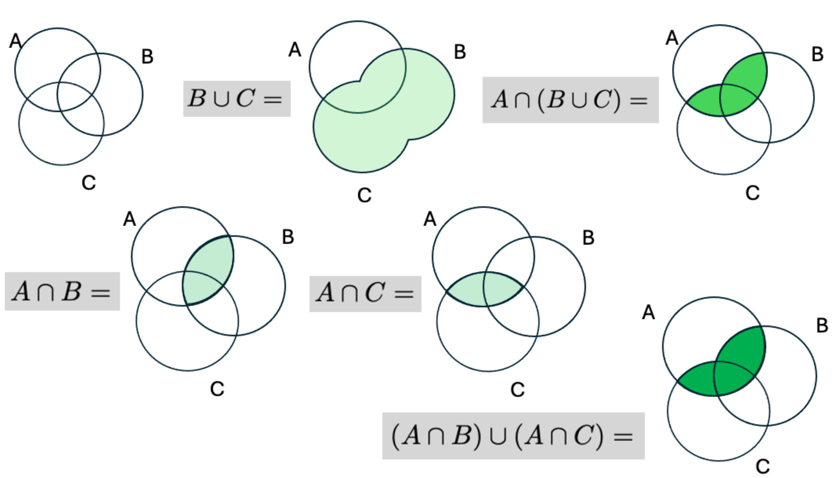
a) Let $x \in A \cap (B \cup C)$. $\Rightarrow x \in A \wedge (x \in B \vee x \in C)$.
1st case: $x \in B \wedge x \in A \Rightarrow x \in A \cap B \Rightarrow x \in (A \cap B) \cup (A \cap C)$
2nd case: $x \in C \wedge x \in A \Rightarrow x \in A \cap C \Rightarrow x \in (A \cap B) \cup (A \cap C)$
$\Rightarrow$ In either case, $x \in (A \cap B) \cup (A \cap C)$
$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ 1pt

Let $x \in (A \cap B) \cup (A \cap C)$. $\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$
1st case: $x \in A \wedge x \in B$. $\Rightarrow x \in A \wedge x \in B \cup C$
2nd case: $x \in A \wedge x \in C$. $\Rightarrow x \in A \wedge x \in B \cup C$
$\Rightarrow$ In either case, $x \in A \cap (B \cup C)$.
$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Hence, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 1pt

Exactly analogous arguments prove the second set equality. 1pt

b) The first equality can be illustrated as follows: 1pt



The second equality can be illustrated analogously. 1pt

NB: These drawings don't prove anything, they just make the result plausible. In order to graphically prove the statements one would have to consider all possible overlapping situations.