I.1.3 Mappings (3 pts)

Are the following $f : X \to Y$ true mappings? If so, are they surjective, or injective, or both? If not, can the pre-image set X be redefined to make X a mapping, and is the resulting mapping surjective, or injective, or both?

a) $X = Y = \mathbb{Z}, \qquad f(m) = m^2 + 1.$

b) $X = Y = \mathbb{N}, \qquad f(n) = n + 1.$

c) $X = Y = \mathbb{R}, \qquad f(x) = \log x.$

d) $X = Y = \mathbb{R}, \qquad f(x) = e^x.$

Solution

a) f is a mapping. It is neither surjective (since $f(n) \geq 1 \ \forall b \in \mathbb{Z}$) nor injective (since $f(-n) = f(n) \ \forall n \in \mathbb{Z}$). 0.5 pt

b) f is a mapping. It is not surjective ($1 \in \mathbb{N}$ has no pre-image). It is injective, since it is monotonic. 0.5 pt

c) f is not a mapping, since $x \leq 0$ has no image. 0.5 pt
However, if we make $X = \{x; x \in \mathbb{R} \wedge x > 0\}$, then f is a mapping that is both surjective (since for every $y \in \mathbb{R}$ there is a real $x > 0$ such that $\log x = y$), and injective (since log is monotonic.). 1 pt

d) f is a mapping, since e^x is defined $\forall x \in \mathbb{R}$. It is not surjective, since $f(x) > 0 \, \forall x \in \mathbb{R}$. It is injective, since it is monotonic. 0.5 pt