0.2.5. **Massive scalar field**

Consider the Lagrangian density for a massive scalar field from the example in ch. 0 §2.5.

a) Generalize this Lagrangian density to a complex field $\phi(x) \in \mathbb{C}$:

$$\mathcal{L} = \frac{1}{2}\,(\partial_\mu \phi(x))\,(\partial^\mu \phi^*(x)) - \frac{m^2}{2}\,|\phi(x)|^2$$

with ϕ^* the complex conjugate of ϕ. What are the Euler-Lagrange equations now?

b) Consider a local gauge transformation, $\phi(x) \to \phi(x)\, e^{i\Lambda(x)}$, with $\Lambda(x)$ a real field that characterizes the transformation. Is the Lagrangian from part b) invariant under such a transformation?

(2 points)

Q.2.5.) a) Treat $\phi(x)$ and $\phi^*(x)$ as independent fields

Minimizing with respect to ϕ^* yields

$$(\partial_\mu \partial^\mu + m^2)\,\phi(x) = 0$$

and minimizing with respect to ϕ just yields the c.c.

$$(\partial_\mu \partial^\mu + m^2)\,\phi^*(x) = 0$$

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b) Under $\phi(x) \to \phi(x)\, e^{i\Delta(x)}$ we have

$$|\phi(x)|^2 \to |\phi(x)|^2$$

and

$$\partial_\mu \phi(x) \to (\partial_\mu \phi(x))\, e^{i\Delta(x)} + i(\partial_\mu \Delta(x))\, \phi(x)\, e^{i\Delta(x)}$$

$$\partial^\mu \phi^*(x) \to (\partial^\mu \phi^*(x))\, e^{-i\Delta(x)} - i(\partial^\mu \Delta(x))\, \phi^*(x)\, e^{-i\Delta(x)}$$

$$\Rightarrow \partial_\mu \phi(x)\, \partial^\mu \phi^*(x) \to \partial_\mu \phi(x)\, \partial^\mu \phi^*(x) - i(\partial^\mu \Delta(x))(\partial_\mu \phi(x))\, \phi^*(x)$$
$$+ i(\partial_\mu \Delta(x))(\partial^\mu \phi^*(x))\, \phi(x)$$
$$+ (\partial_\mu \Delta(x))(\partial^\mu \Delta(x))\, |\phi(x)|^2$$

$$+ \partial_\mu \phi\, \partial^\mu \phi^*$$

1 $\to$ The Lagrangian is not invariant