0.3.2. **Harmonic oscillator coupled to a magnetic field**

Consider a charged 3-d classical harmonic oscillator (oscillator frequency ω_0, charge e). Put the oscillator in a homogeneous time-independent magnetic field $\boldsymbol{B} = (0, 0, B)$. Show that the motion remains oscillatory, and find the oscillation frequencies in the directions parallel and perpendicular, respectively, to $\boldsymbol{B}$.

(4 points)

0.3.2 ·) In addition to the restoring force $-m\omega_0^2 x$, the particle is subject to a Lorentz force

$$\frac{e}{c}\vec{v}\times\vec{B} = \frac{e}{c}(\dot{y}B, -\dot{x}B, 0)$$

→ The eqs of motion are

$$\ddot{x} + \omega_0^2 x = \Omega\dot{y} \qquad (1)$$

$$\ddot{y} + \omega_0^2 y = -\Omega\dot{x} \qquad (2)$$

$$\ddot{z} + \omega_0^2 z = 0 \qquad (3)$$

1 with $\Omega := eB/mc$ the cyclotron frequency.

(3) → For oscillations in the z-direction, the frequency $\omega = \omega_0$ is unchanged

1

Define $\xi := x + iy$ and write

(1) + i·(2) → $\boxed{\ddot{\xi} + \omega_0^2\xi = -i\Omega\dot{\xi}}$

1

ansatz: $\xi(t) = \xi_0 e^{i\omega t}$

→ $-\omega^2 + \omega_0^2 =$

$$\omega^2 + \Omega\omega - \omega_0^2 = 0$$

→ $\omega = \frac{1}{2}\left(-\Omega \pm \sqrt{\Omega^2 + 4\omega_0^2}\right) = \pm\sqrt{\omega_0^2 + \Omega^2/4} - \Omega/2$

→ The motion in the x-y plane is oscillatory, and eigenfrequencies are

1 $$\omega_\pm = \sqrt{\omega_0^2 + \Omega^2/4} \pm \Omega/2$$