1.1.1. **Dual field tensor**

Show that the dual field tensor $\tilde{F}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\lambda\kappa}F_{\lambda\kappa}$ obeys $\partial_\mu \tilde{F}^{\mu\nu}(x) = 0$.

hint: First show that $\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu} = 0$, and then relate $\partial_\mu \tilde{F}^{\mu\nu}(x)$ to that expression.

(2 points)

1.1.1.) Ch. 1 $1.1 def. 1 $\rightsquigarrow \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$\rightsquigarrow \quad \partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu} = \partial^\lambda\partial^\mu A^\nu - \partial^\lambda\partial^\nu A^\mu + \partial^\mu\partial^\nu A^\lambda - \partial^\mu\partial^\lambda A^\nu + \partial^\nu\partial^\lambda A^\mu - \partial^\nu\partial^\mu A^\lambda = 0 \quad (*)$$

1

$$\rightsquigarrow \quad \partial_\mu \tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}\partial_\mu F_{\alpha\beta}$$

$$\overset{\text{relabel}}{=} \frac{1}{3}\left[\varepsilon^{\mu\nu\alpha\beta}\partial_\mu F_{\alpha\beta} + \varepsilon^{\alpha\nu\beta\mu}\partial_\alpha F_{\beta\mu} + \varepsilon^{\beta\nu\mu\alpha}\partial_\beta F_{\mu\alpha}\right]$$

ε antisymmetric

$$= \frac{1}{3}\varepsilon^{\mu\nu\alpha\beta}\left[\partial_\mu F_{\alpha\beta} + \partial_\alpha F_{\beta\mu} + \partial_\beta F_{\mu\alpha}\right] \overset{(*)}{=} 0$$

1

$\square$