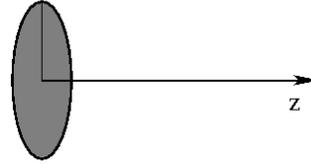
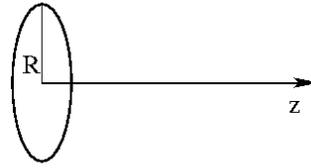


2.2.1. Planar charge distributions

- a) Consider a homogeneously charged infinitesimally thin ring with radius R and total charge Q that is oriented perpendicular to the z -axis. Calculate the electric field on the z -axis.
- b) The same for a homogeneously charged disk with charge density σ and radius R . Consider the limits $z \rightarrow \infty$, $z \rightarrow 0$, and $R \rightarrow \infty$, and ascertain that they makes sense.



(4 points)

2.2.1-1.) c) Let the charge be in the $z=0$ plane.

$$\rho(\vec{r}) = \rho_0 \delta(y \pm z) \delta(r - R)$$

~ cylindrical coordinates.

Total charge: $\int d\vec{r} \rho(\vec{r}) = 2\pi \rho_0 =: Q$

Poisson's formula

$$\varphi(\vec{x}) = \int d\vec{r} \frac{\rho(\vec{r})}{|\vec{x} - \vec{r}|}$$

electric field: $\vec{E} = -\vec{\nabla} \varphi = -\int d\vec{r} \rho(\vec{r}) \vec{\nabla} \frac{1}{|\vec{x} - \vec{r}|} = \int d\vec{r} \rho(\vec{r}) \frac{\vec{x} - \vec{r}}{|\vec{x} - \vec{r}|^3}$

symmetry $\rightarrow \vec{E}(\vec{x} = (0, 0, z)) = E(z) \hat{z}$

$\rightarrow \underline{E(z)} = z \int d\vec{r} \frac{\rho(\vec{r})}{|\vec{x} - \vec{r}|^3} = z \int_0^{2\pi} d\varphi \int_0^{2\pi} \frac{\rho_0}{(z^2 + R^2)^{3/2}} = \frac{Qz}{(z^2 + R^2)^{3/2}}$

(1)

b) Charge density: σ

\rightarrow Charge on ring with radius r , thickness dr :

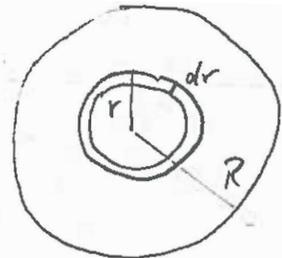
$$dQ = \sigma 2\pi r dr = \frac{Q}{\pi R^2} 2\pi r dr = \frac{2Q}{R^2} r dr \quad \underline{Q = \sigma \pi R^2}$$

c) $\rightarrow \underline{E(z)} = \int_0^R dr \frac{2Q}{R^2} r \frac{z}{(z^2 + r^2)^{3/2}} = \frac{2Q}{R^2} z \int_0^R \frac{dx}{(z^2 + x^2)^{3/2}}$

$$= \frac{2Q}{R^2} \left[\frac{1}{(1+x^2)^{1/2}} \right]_0^{R^2/z^2} = \frac{2Q}{R^2} \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right) = 2\sigma \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right)$$

(1)

$\underline{E(z \rightarrow \infty)} = \frac{2Q}{R^2} \left(1 - 1 + \frac{1}{2} \frac{R^2}{z^2} + O\left(\frac{R^4}{z^4}\right) \right) = \underline{Q/z^2} + O(z^{-4})$ field of point charge.



$$\underline{E(z \rightarrow 0) = E(R \rightarrow \infty) = 25\text{V}}$$

In infinite sheet will wasted energy during production a field

(1) Let's independent of z !