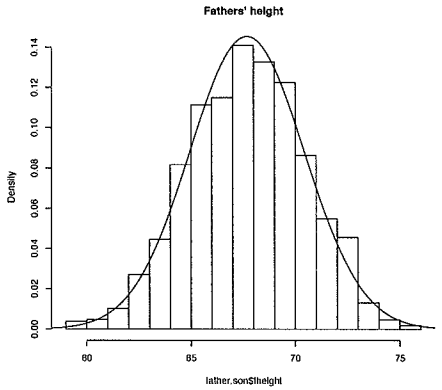
Density curves



Smooth curve approximates the histogram.
(Use *frequencies* in histogram.)

Pearson's Father-Son Data
Frequency
0
50
100
150
60
65
70
75
Fathers' heights

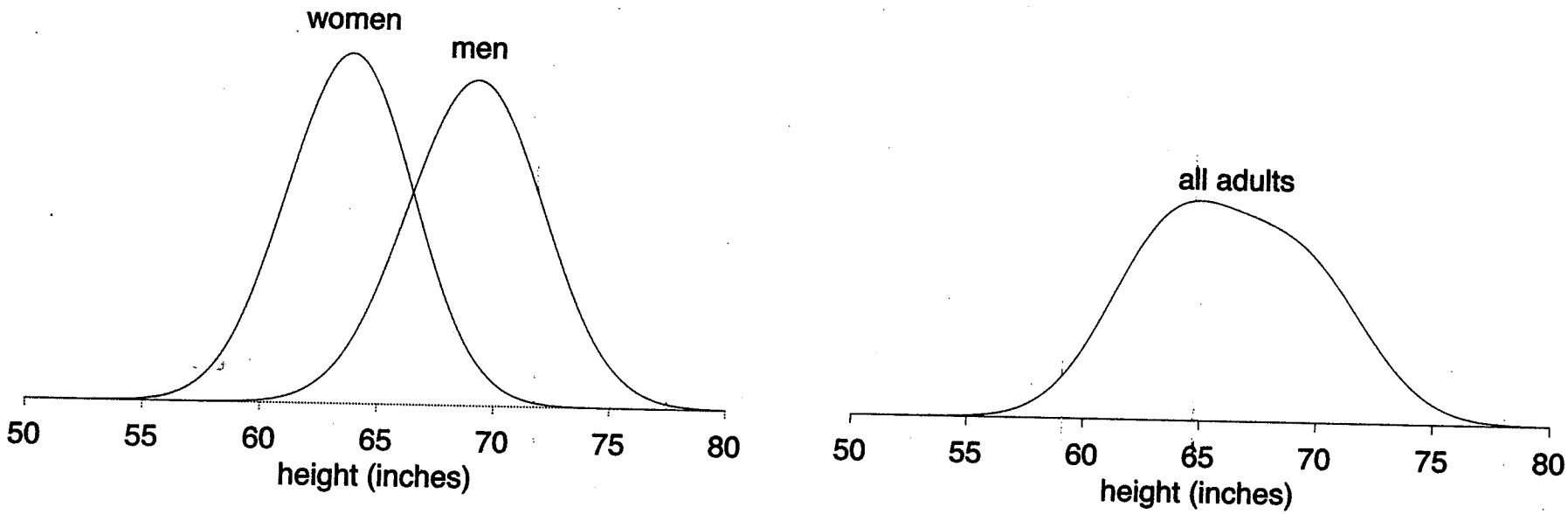
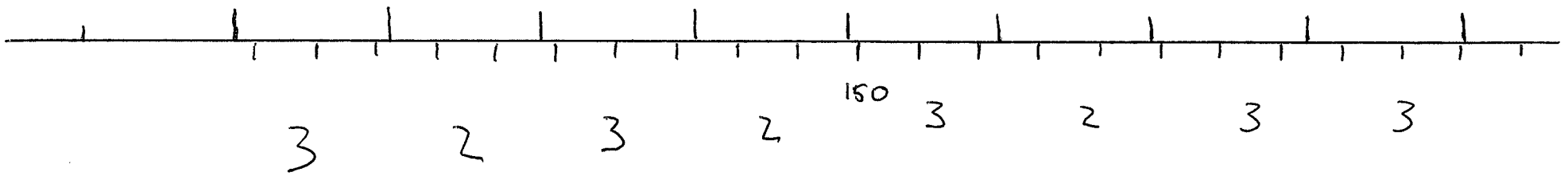


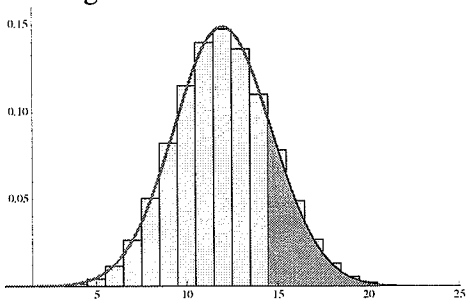
Fig. 3.9 (a) Distributions of heights of adult men and women in the United States. (b) Distribution for all adults (52% women, 48% men). The combined distribution has only one mode because the mean heights of men and women are so close together.

58
60
62
64
66
68
70
Height (inches)

59 inches



If a density curve approximates a histogram, the proportion of observations falling in a range is the area under the curve and above the range.



Normal Curve

A normal distribution has a bell-shaped density curve. It has two parameters, **mean** and **standard deviation**.

68-95-99.7 Rule: A Normal distribution with mean μ and standard deviation σ satisfies:

- Approximately 68% of observations are within σ of mean μ
- Approximately 95% are within 2σ of μ
- Approximately 99.7% are within 3σ of μ.

Example

The distribution of heights of women aged 20 to 29 is approximately Normal with mean 64 inches and standard deviation 2.7 inches.
Questions:

- Between what heights do the middle 95% of young women fall?
- What percent of young women are taller than 61.3 inches?

Standard normal distribution

If x is an observation from a distribution that has mean μ and standard deviation σ, the standardized value of x is

$$z = \frac{x - \mu}{\sigma}.$$

A standardized value is often called a z-score.

The **standard Normal distribution** is the Normal distribution with mean 0 and standard deviation 1.

If a variable x has any Normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ, then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

has the standard Normal distribution

Exercise

The heights of women aged 20 to 29 are approximately Normal with mean 64 inches and standard deviation 2.7 inches. Men the same age have mean height 69.3 inches with standard deviation 2.8 inches. What are the z-scores for a woman 6 feet tall and a man 6 meet tall?

What information does the z-score convey that the actual heights do not?

Problem 3.45

The heights of women aged 20 to 29 follow approximately the $N(64, 2.7)$ distribution. Men the same age have heights distributed as $N(69.3, 2.8)$. What percent of young men are shorter than the mean height of women?

The cumulative proportion for a value x in a distribution is the proportion of observations in the distribution that lie at or below x.

How hard do locomatives pull? An important measure of the performance of a locomtovie is its "adhesion", which is the locomotive's pulling force as a multiple of its weight. The adhesion of one 4400-horespower locomotive model values in actual use accordinate to a Normal distribution with mean $\mu = 0.37$ and standard deviation $\sigma = 0.05$.

- What proportion of adhesions measured in use are higher than 0.40?
- What proportion of adhesions are between 0.40 and 0.50?

Problem 3.28

Scores on the Wechsler Adult Intelligence Scale (WAIS) are approximately Normal with mean 100 and standard deviation 15. People with WAIS scores below 70 are considered mentally retarded when, for example, applying for Social Security disability benefits. According to the 68-95-99.7 rule, about what percent of adults are retarded by this criterion?