Errata for *Markov Chains and Mixing Times*

Note: This list is not comprehensive. An updated second edition is freely available at `http://pages.uoregon.edu/dlevin/MARKOV`.

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[BFJSZ below is the Stanford reading group consisting of Bormashenko, Fadnavis, Jiang, Shender and Zhu. BFMM are the IMPA readers Brito, Franco, Misturini and Morgado.]

Introduction

Page xiv, last line [ELW, 7/15/2010] (E) Change 17 to 16.

Chapter 1: Introduction to Finite Markov Chains

Page 6, first display [A. Holroyd, 1/9/2009] (D) This should be

$$\mu(A) = \sum_{x \in A} \mu(x).$$

Page 12, Proposition 1.14 [F. Nazarov, 3/19/09] (D) Part (ii) requires the uniqueness of the stationary distribution, which is proven in Section 1.5.4, immediately following Proposition 1.14. Therefore, it should be removed from Proposition 1.14, and the following should be added after Corollary 1.17:

Proposition. *If π is the unique solution to $\pi = \pi P$ for an irreducible transition matrix P, then $\pi(x) = 1/\mathbf{E}_x \tau_x^+$.*

Proof. Let $\tilde{\pi}_z(y)$ equal $\tilde{\pi}(y)$ as defined in (1.19), and write $\pi_z(y) = \tilde{\pi}_z(y)/\mathbf{E}_z \tau_z^+$. Proposition 1.14 implies that π_z is a stationary distribution, and Corollary 1.17 implies π_z does not depend on z. Thus if $\pi(y) := \pi_z(y)$ for some (and hence all z), then

$$\pi(x) = \pi_x(x) = \frac{\tilde{\pi}_x(x)}{\mathbf{E}_x \tau_x^+} = \frac{1}{\mathbf{E}_x \tau_x^+} .$$

■

Page 13, Equation (1.27) [F. Nazarov, 3/19/09] (D) This should be

$$\mathbf{P}_{x_0}\{(X_{\tau+1}, X_{\tau+2}, \ldots, X_{\tau+\ell}) \in A \mid \tau = k \text{ and } (X_1, \ldots, X_k) = (x_1, \ldots, x_k)\} \\ = \mathbf{P}_{x_k}\{(X_1, \ldots, X_\ell) \in A\}, \quad (1.27)$$

Page 14, middle of proof of Corollary 1.17 [J. Steiff 11/12] (D) Remove “square”

Page 15, Example 1.20 [P. Prałat, 7/7/2010] (D) The right-hand side of the display should be $\pi(y)P(y,x)$ instead of $\pi(y)P(x,y)$.

Page 16, Section 1.7 [P. Prałat, 7/7/2010] (D) The definition of $x \leftrightarrow y$ should be that $x \to y$ and $y \to x$, or $x = y$, making $\leftrightarrow$ an equivalence relation.

Page 19, Exercise 1.12 (D) The statement of the exercise is incorrect. The exercise should say:

Prove that there exists $y \in B$ with $h(y) = \max_{x \in \Omega} h(x)$.

Page 19, hint to Exercise 1.13 [P. Sousi, 7/11/2010] (E) $P(x,y)$ should be $P(y,x)$.

Page 19, Exercise 1.14 [P. Sousi, 7/11/2010] (E) $P(x,y)$ should be $P(y,x)$, and replace “the” by “a”.

Page 19, Exercise 1.14 [P. Sousi, 7/11/2010] (E) This is nearly identical to Exercise 1.9.

Chapter 2: Classical (and Useful) Markov Chains

Page 23, Proposition 2.4, [P. Prałat, 4/27/2011] (D) Here, and throughout the text log regers to natural logarithm.

Page 25, Lemma 2.5, [P. Prałat, 4/27/2011] (D) $[X_t]$ should be replaced by $([X_t])$.

Page 25, proof of Lemma 2.6, [P. Prałat, 4/27/2011] (D) Consider the infinite sequence $U_0, U_1, U_2, \ldots$ of independent and identically distributed random variables.

Page 25, proof of Lemma 2.6, [P. Prałat, 4/27/2011] (D) Replace "is one of the j+1 smallest" by "is one of the j smallest".

Page 27, first displayed equation [P. Prałat, 7/7/2010] (D) For notational consistency, the left-hand side should be $\tilde{\pi}(k)$, not $\tilde{\pi}_k$.

Page 29, first displayed equation [P. Prałat, 7/7/2010] (D) $P^m(a,b)$ should be replaced by $P^r(a,b)$.

Page 33, Theorem 2.24 [P. Prałat, 7/7/2010] (D) The first sentence should be "...of which exactly b steps go up."

Page 34, Exercise 2.3 [P. Prałat, 7/7/2010] (D) The solution given in Appendix C applies not to the walk described in the problem, but rather to the walk which moves deterministically from n to $n-1$.

Page 34, Exercise 2.5 [P. Prałat, 7/7/2010] (D) In the solutions in Appendix C, d should be replaced by n. Moreover, two extreme cases should be considered. For $y = 0$, we have

$$\sum_{x=0}^{n} \pi(x)P(x,0) = \pi(1)P(1,0) = \binom{n}{1} 2^{-n} \frac{1}{n} = 2^{-n} = \pi(0)\,.$$

For $y = n$, we have

$$\sum_{x=0}^{n} \pi(x)P(x,n) = \pi(n-1)P(n-1,n) = \binom{n}{n-1} 2^{-n} \frac{1}{n} = 2^{-n} = \pi(n)\,.$$

Page 34, Exercise 2.8 [P. Prałat, 7/7/2010] (D) In the solutions in Appendix C, on the right-hand side of equation C.4, $\hat{P}(w,z)$ should be replaced by $\hat{P}(\varphi(z), \varphi(w))$.

Chapter 3: Markov Chain Monte Carlo: Metropolis and Glauber Chains

Page 41, definition of dynamics for hardcore model [P. Prałat, 7/7/2010] (D) The second bullet should end with "... while if no adjacent vertex is occupied, v is occupied by a particle with probability 1/2, and is vacant otherwise."

Page 43, before second displayed equation [P. Prałat, 7/7/2010] (D) Replace $\mathcal{N}$ by $\mathcal{N}(w)$ in "Denote the set of occupied neighbors of w by $\mathcal{N}$...".

Page 44, equation 3.11 [BFJSZ, 3/28/10] (E) Sum over $w \in V$ on the right-hand side.

Chapter 4: Introduction to Markov Chain Mixing

Page 47, last display [S. Janson, 5/7/10] (D) $|\dots|$ missing 3 times.

Page 47, (4.1) [P. Prałat, 2010] (D) Perhaps clearer if we write

$$\|\mu - \nu\|_{\text{TV}} = \max_{A \subseteq \Omega} |\mu(A) - \nu(A)|,$$

, although note that we do not intend the symbol $\subset$ to mean strict subset; for that we use $\subsetneq$.

Page 48, caption for Figure 4.1, [P. Prałat, 2010] (D) To be consistent with the proof of Proposition 4.2, B should be defined as $B = \{x : \mu(x) \geq \nu(x)\}$.

Page 51, after (4.12) [P. Sousi, 7/30/11] (E) The intuition is misleading. X is chosen in the union of regions I and III. If it is in III then Y=X and if it is in I then Y is chosen independently in region II. The current version is wrong because the union of the three regions has area greater than 1.

Page 55, line 1 [P. Sousi, 5/11/10] (E) What is used here is Lemma 4.11, and not the non-increasing property.

Page 55, 2u [S. Janson, 5/7/10] (D) $X_k = g_k g_{k-1} \cdots g_1$ should replace $X_k = g_1 g_2 \dots g_k$.

Page 56, 4d (D) $a_t a_{t-1} \dots a_1 = a$ should replace $a_1 a_2 \cdots a_t = a$.

Page 57, equation 4.39 (D) This should be

$$\widehat{P}(n, n) = \widehat{P}(n, n-1) = 1/2. \tag{4.39}$$

Page 57, last line [S. Janson, 5/7/10] (D) $\varepsilon < 1/4$ should replace $\varepsilon \leq 1/4$.

Page 59, Exercise 4.2, [P. Prałat, 2011] (D) The sequence $\{n_k\}$ should satisfy $n_k \to \infty$ as $k \to \infty$.

Chapter 5: Coupling

Page 64, equation 5.2 (D) The statement that any coupling can be modified so that it remains together after the first collision is false. This is true for *Markovian* couplings, which are couplings $(X_t, Y_t)_{t \geq 0}$ such that if $Z_t = (X_t, Y_t)$, then $(Z_t)_{t \geq 0}$ is a Markov chain.

Page 65, 6u [P. Prałat, 1/6/2011] (D) The constant c_1 is positive.

Page 65, 2nd to last line [J.!Steif, 11/2012] (D) First n should be d.

Page 66, Theorem 5.5 [P. Prałat, 1/6/2011] (D) One needs $\varepsilon < \frac{1}{2}$.

Page 67, last inequality in proof of Theorem 5.5 [Y. Abe, 2/17/13] (D) $\log(\varepsilon^{-1})$ should be $\log_2(\varepsilon^{-1})$.

Page 69, end of subsection 5.3.5 [P. Prałat, 1/6/2011] (D) The text should read "...the mixing time of the reversed winning streak was $O(n)$."

Page 69, Proof of Proposition 5.6, [P. Prałat, 1/6/2011] (D) Note that $Z_0 = x$ and $X_0 = x$.

Page 69, above of last display [BFMM] (D) The sum in the definition of N_t should be indexed by s instead of t.

Page 71, 3 lines above (5.16) [J. Steif, 11/2013] (D) The statement should be that the number of disagreements *may* increase. If the color of w already was $x(v_0)$, then a new disagreement is not introduced.

Page 73, [P. Sousi] (D) Here define

$$X_{t-1} = \begin{cases} Z_{t-1-(N_t - W_1)} & \text{if } t \leq \tau \\ Y_t & \text{if } t > \tau, \end{cases}$$

where τ is the first time that $t - 1 - (N_t - W_1)$ equals N_t. Note that τ is the first time that $2(N_t - W_1) - (t-1) = -W_1$. Observe that $2(N_t - W_1)$ is a simple random walk for $t \geq 1$.

Page 74, Exercise 5.3 [J. Steig, 11/2013] (D) Add $\mathbf{E}(tau) < \infty$ to the conditions.

Chapter 6: Strong Stationary Times

Page 79, 2d [P. Prałat, 3/24/11] (D) Replace "$B \subset \Omega^s$" by "$B \subset \Lambda^s$". In the following sentence the function $\tilde{f}_r$ should be defined on $\Omega \times \Lambda^r$, i.e. the text should read $\tilde{f}_r : \Omega \times \Lambda^r \to \Omega$. Also, the second displayed equation should read

$$\begin{aligned}\mathbf{P}_x\{X_t = y \mid \tau = s,\, X_s = z\} \\ &= \mathbf{P}_x\{\tilde{f}_{t-s}(z, Z_{s+1}, \ldots, Z_t) = y \mid (Z_1, \ldots, Z_s) \in B,\, X_s = z\} \\ &= P^{t-s}(z, y).\end{aligned}$$

Page 81, 13u [BFJSZ, 3/28/10] (E) Replace k/n with $(k+1)/n$.

Page 83, 3u [A. Frieze, 3/11/10] (E) Replace "right-hand side" by "left-hand side".

Page 84, Exericse 6.4 [P. Prałat, P. Sousi, 3/24/11] (D) The exercise should read:

(a) Show that for each $t \geq 1$ there is a stochastic matrix Q_t so that $P^t(x, \cdot) = [1 - s(t)]\,\pi + s(t)Q_t(x, \cdot)$ and $\pi = \pi Q_t$.

(b) Using the representation in (a), show that

$$P^{t+u}(x, y) = [1 - s(t)s(u)]\,\pi(y) + s(t)s(u) \sum_{z \in \Omega} Q_t(x, z)Q_u(z, y). \tag{6.19}$$

(c) Using (6.19), establish that s is submultiplicative: $s(t + u) \leq s(t)s(u)$.

Page 84, Exercise 6.5 [BFJSZ, 3/28/10] (E) Add the assumption that τ is a strong stationary time.

Page 84, Exercise 6.5 [P. Prałat, 3/24/11] (D) Replace t/t_0 by $\lfloor t/t_0 \rfloor$.

Chapter 7: Lower Bounds on Mixing Times

Page 88, last line [S. Janson, 5/7/10] (D) 2 should be 1/2.

Page 90, equation (7.16) [A. Frieze, 3/11/10] (E) Replace the μ_s by μ_S. The equation should now read

$$\frac{1}{2} \leq \|\mu_S - \pi\|_{\mathrm{TV}} \leq \left\|\mu_S - \mu_S P^t\right\|_{\mathrm{TV}} + \left\|\mu_S P^t - \pi\right\|_{\mathrm{TV}}.$$

Page 90, after (7.16) [BFMM, 8/8/11] (D) This should read "Taking $t = t_{\mathrm{mix}} = t_{\mathrm{mix}}(1/4)$ in (7.16), by Exercise 4.1 and the inequality in (7.15),

..."

Page 90, example 7.4, [S. Janson, 5/7/10, also P. Sousi, 5/12/10] (D) We should have $\sum_{x \in V_1} \deg(x) = 2dn^d + 2d$ (instead of $2dn^2 + 2d$). Therefore, the displayed equation should read

$$\Phi_\star \le \Phi(V_1) = \frac{(2d)}{2[2d(n^d+1)]} \le \tfrac{1}{2} n^{-d}.$$

As well, the lower bound on the next line should be $n^2/2$.

The correct order of t_{mix} is $n^2 \log n$ only when $d = 2$.

Page 90, example 7.4, [P. Prałat, 3/24/11] (D) The text should read $V_1 \cap V_2 = \{v^\star\}$.

Page 91, display following (7.17), [J. Steif, 11/13] (D) The first "=" is an "$\le$".

Page 91, example 7.7 [S. Janson, 5/7/10] (D) The bound from Section 5.3.4 is $16n$. We now get $t_{\text{mix}} \ge (n-2)/2$.

Page 92, second displayed equation, and following [S. Janson, 5/7/10] (D) Since the walk is lazy, the equation should be

$$Q(S, S^c) = \pi(v_r) P(v_r, v_0) = \left(\frac{3}{2n-2}\right) \frac{1}{6} = \frac{1}{4(n-1)}.$$

Thus the following line should have $\Phi(S) = \frac{1}{2(n-2)}$ and the following displayed equation should read

$$t_{\text{mix}} \ge \frac{1}{4\Phi_\star} \ge \frac{1}{4\Phi(S)} = \frac{n-2}{2} = \frac{2^{k+1}-3}{2}.$$

Also, on page 91, 4 lines from bottom, the sentence should read "We now show that $t_{\text{mix}} \ge (n-2)/2$."

Page 93, equation 7.20 [P. Prałat, 3/24/2011] (D) The right-hand side of the equation should be $\left(\sum_{x \in \Lambda} x^2 \eta(x)\right) \left(\sum_{x \in \Lambda} [r(x) - s(x)]^2 \eta(x)\right)$.

Page 95, the line before (7.27), [BFMM, 8/8/11] (D) "By Proposition 7.8" should be "By Remark 7.11".

Page 96, second displayed equation, [BFMM, 8/8/11] (D) The penultimate bound should be $2\mathbf{P}\{S_{\alpha n^2} > n/4\}$.

Page 97, last display, [BFMM, 8/8/11] (D) The last display should be

$$|P^{t(n,\alpha)}(x_0, A) - \pi(A)| \ge \frac{1}{2} - \frac{1}{\alpha^2}.$$

Pg 98, first line, [J. Steif, 11/13] (D) Sum goes from 2 to $n+1$.

Page 98, Exercise 7.1 [P. Prałat, 3/24/2011] (D) Replace $\mathrm{Var}(W(X_t))$ by $\mathrm{Var}(W(\mathbf{X}_t))$.

Chapter 8: The Symmetric Group and Shuffling Cards

Page 102, first bullet [J. Steiff] (D) Remove clause "and the same card occupies position Y_t in both decks".

Page 101, 2d [R. Pinsky, 9/3/09] (D) The displayed equation should be

$$(a_1 a_2 \dots a_m) = (a_1 a_2)(a_2 a_3) \cdots (a_{m-1} a_m) .$$

Page 101, equation 8.3, [P. Sousi, 7/11/10] (D) σ should replace ρ on the right-hand side.

Page 103, first display [P. Sousi, 7/11/10] (D) The conditioning is not needed.

Page 106, 4d [P. Sousi, 7/11/10] (D) For consistency, P_n^t should be P^t. (Although note that P^t does in fact depend on n.)

Chapter 9: Random Walks on Networks

Page 117, 1u [R. Pinsky, 9/3/09] (D) For notational consistency, replace $c(x,y)$ by $c(\vec{xy})$.

Page117, 5d [J. Steif] (D) Replace $-h$ by $-g$.
Page 118, 10d [A. Frieze, 3/23/10] (E) Replace n by m in the parenthetical comment, so that we have "(i.e., for some $x_0, \dots, x_{m-1} \in V$ we have $\vec{e_i} = (x_{i-1}, x_i)$, where $x_m = x_0$)".

Page 119, Proposition 9.5 [S. Janson, 5/7/10] (D) The statement should begin: "For any $a, z \in \Omega$ with $a \neq z$".

Page 120, 2d (D) The identity should read "$I(\overrightarrow{v_1 v_2}) := I(\overrightarrow{v_1 v}) + I(\overrightarrow{v v_2})$".

Page 120, 5d [R. Pinsky, 9/3/09] (D) Replace "potentials" by "voltages".

Page 120, Example 9.8 [P. Prałat, 4/27/11] (D) Instead of $|e| = i$, the text should say "if the vertex of e closest to the root is at distance i to the root".

Page 121, proof of Theorem 9.10 [S. Janson, 5/7/10] (D) The set of unit-strength flows is not bounded. But the set with energy bounded by a constant C is compact, which suffices.

Page 122, 6u [S. Janson, 5/7/10] (D) Replace $\Pi \subseteq V$ with $\Pi \subseteq E$.

Page 123, figure 9.1 [P. Sousi, 10/17/2009] (E) Cutset shown is Π_4.

Chapter 10: Hitting Times

Page 128, display above (10.4) [S. Janson, 5/7/10] (D) $\mathbf{E}_\pi$ should be $\mathbf{E}_a$

Page 129, 5u [R. Pinsky, 9/3/09, J. Steif] (D) The term $+1$ should be $+2$.

Page 130, Lemma 10.5 [P. Sousi, 10/17/2009, J. Steif]] (E) In Lemma 10.5 we should add a condition that $\mathbf{E}_a(\tau) < \infty$.

Page 131, Lemma 10.10 and Remark 10.11 [S. Janson, 5/7/10] (D) This is correct, but it should be noted that in the proof of Proposition 10.9 we really use the generalization to arbitrary many states.

Page 132, second line of Proof [S. Janson, 5/7/10, J. Steif] (D) u and v should be x and y.

NOT FIXED **Page 133, Proposition 10.13 [R. Pinsky, 9/3/09; P. Prałat, 4/27/11] (D)** The upper bound when $d \geq 3$ is not proven in the text, but is discussed in Exercise 9.1. The effective resistance between opposing corners of the d-dimensional box can be estimated via a multi-type Pólya's urn process, similar to the case $d = 2$. Details for the case $d = 3$ can be found in the paper "Pólya's theorem on random walks via Pólya's urn", by D.A. Levin and Y. Peres, to appear in *American Mathematical Monthly*; also available at `http://www.uoregon.edu/~dlevin/polya.pdf`.

Page 133, Equation (10.17), [ELW, 7/15/10, J. Steif] (E) This should be

$$\mathbf{E}_a(\tau_b) = dn^d \mathcal{R}(a \leftrightarrow b). \tag{10.17}$$

Page 134, Remark 10.15, last line [1/11/2009] (D) The phrase "time to" should be "probability at"

Page 135, Proposition 10.18, [P. Sousi, 8/8/2011] (D) In the proof of part (ii), the construction of the enlarged graph is not always right. If $P_L(x,x) = 1/2$ then $P(x,x) = 0$ and we do not create a state m_{xx} at all. If $P_L(x,x) > 1/2$ then we do define m_{xx} but assign it stationary measure $\pi_K(x,x) = \pi(x)P(x,x)/2$.

Page 137, third display, [J. Steif] (D) Replace 1 by $\pi(y)$ inside square brackets.

Page 138, Equation 10.29 [A. Frieze, 4/4/10] (D) Middle expectation should be $\mathbf{E}_{x,y}$.

Pg 138, first line of Proof, [J. Steif] (D) Define $\varphi(v_\star, i) = 1$, say.

Page 139, Corollary 10.22 [R. Pinsky, 9/3/09, P. Sousi 7/11/2010] (D) The statement should be for $d = 2$. The lower bound is not proven in the text, and should be omitted from the statement.

Page 140, Exercise 10.9, line 3 (D) $\sum_{k=0}^{\infty} b_k s^l$ should be $\sum_{k=0}^{\infty} b_k s^k$.

Page 140, exercise 10.12 [P. Sousi, 3/7/11] (E) The 6 should be changed to 7.

Page 140, exercise 10.13 [P. Sousi, YP 3/7/11] (E) Exercise 10.13 is wrong – one can find counterexamples. (Even a 2 state chain is a counterexample.)

Page 141, Notes, line 1 (D) This sentence should be moved to the Notes of the next chapter.

Chapter 11: Cover Times

Page 145, 5d [P. Sousi, 7/11/10] (D) The expression $T_k - T_{k-1}$ should be $\mathbf{E}_x(T_k - T_{k-1})$.

Page 145, 8d and Page 146, 15u [R. Pinsky, 9/3/09] (D) Exercise 11.1 is not needed.

Page 151, second paragraph, proof of Prop. 11.9 [BFJSZ, 3/29/10] (E) Should read "$|B^c| \leq 2 + 4 + \cdots + 2^j$".

Chapter 12: Eigenvalues

Page 158, 4u [R. Pinsky, 9/3/09] (D) Here ω^{2n-1} should be $\omega^{2(n-1)}$.

Page 160, 1u [P. Sousi, 7/11/10] (D) λ_j should be a $\lambda^{(j)}$.

Page 161, equation 12.20, [P. Sousi, 7/11/10] (D) P_j should be P_i.

Page 167, Exercise 12.1(b) [BFJSZ, 3/28/10] (E) Should read "shows that gcd $\mathcal{T}(x)$ does not depend on x."

Chapter 13: Eigenfunctions and Comparison of Chains

Page 179, 2d [BFJSZ, 3/28/10] (E) Should read "which holds because f is non-negative everywhere."

Chapter 14: The Transportation Metric and Path Coupling

Page 196, Theorem 14.12 [A. Adelmann, 3/9/09] (D) In the statement of the theorem, (14.21) should be the following quantity:

$$2n \left\lceil \frac{n\log(n) + n\log(3nq^n/\varepsilon)}{c(q,\Delta)} \right\rceil \left\lceil \frac{27qn}{\eta\varepsilon^2} \right\rceil . \tag{14.21}$$

Replace the displayed equation before (14.22) by

$$t(n,\varepsilon) = \left\lceil \frac{n\log(n) + n\log(3nq^n/\varepsilon)}{c(q,\Delta)} \right\rceil \geq t_{\mathrm{mix}}\left(\frac{\varepsilon}{3nq^n}\right)$$

and replace (14.22) by

$$\|P^{t(n,\varepsilon)}(x_0,\cdot) - \pi_k\|_{\mathrm{TV}} \leq \frac{\varepsilon}{3nq^n} . \tag{14.22}$$

Replace the right-most quantity in the displayed equation below (14.22) and in (14.23) by $\frac{\varepsilon}{3nq^n}$. Replace the right-most quantities in the displayed equation above (14.26) by $n \cdot \frac{\varepsilon}{3nq^n} = \frac{\varepsilon}{3q^n}$. Replace (14.26) by

$$\mathbf{E}(W) = \frac{1}{|\Omega|} + \tilde{\varepsilon}, \quad \text{where } |\tilde{\varepsilon}| \leq \frac{\varepsilon}{3q^n} . \tag{14.26}$$

Replace the end of the proof following the sentence that begins "By Chebyshev's..." with:

Therefore

$$\mathbf{P}\left\{\left(1 - \frac{\varepsilon}{3}\right)\mathbf{E}W \leq W \leq \left(1 + \frac{\varepsilon}{3}\right)\mathbf{E}W\right\} \geq 1 - \eta,$$

and since $\mathbf{E}W = 1/|\Omega| + \tilde{\varepsilon}$,

$$\mathbf{P}\left\{\left(1 - \frac{\varepsilon}{3}\right)\left(\frac{1}{|\Omega|} + \tilde{\varepsilon}\right) \leq W \leq \left(1 + \frac{\varepsilon}{3}\right)\left(\frac{1}{|\Omega|} + \tilde{\varepsilon}\right)\right\} \geq 1 - \eta.$$

Applying $|\tilde{\varepsilon}| \leq \frac{\varepsilon}{3q^n} \leq \frac{\varepsilon}{3|\Omega|}$ shows that

$$\mathbf{P}\left\{\left(1-\frac{\varepsilon}{3}\right)\frac{1}{|\Omega|}-\frac{\varepsilon}{3}\frac{1}{|\Omega|}+\frac{\varepsilon^2}{9}\frac{1}{|\Omega|}\leq W\leq\left(1+\frac{\varepsilon}{3}\right)\frac{1}{|\Omega|}+\frac{\varepsilon}{3}\frac{1}{|\Omega|}+\frac{\varepsilon^2}{9}\frac{1}{|\Omega|}\right\}\geq 1-\eta.$$

Thus

$$\mathbf{P}\left\{(1-\varepsilon)\frac{1}{|\Omega|}\leq W\leq(1+\varepsilon)\frac{1}{|\Omega|}\right\}\geq 1-\eta.$$

For every step of the Glauber dynamics we need two uniform random variables (one for the vertex and one for the colour). We need a_n samples for every Ω_k, which shows that at most (14.21) uniform variables are required.

Chapter 15: The Ising Model

Page 206, 5u [BFJSZ, 3/28/10] (E) Should read "the parent of v".

Chapter 16: From Shuffling Cards to Shuffling Genes

Page 218, 12u [R. Pinsky, 9/3/09] (D) Change "or" to "of".

Page 220, 10d [R. Pinsky, 9/3/09, BFJSZ, 3/28/10] (E) Change the middle $\geq$ to $\leq$.

Chapter 17: Martingales and Evolving Sets

Page 235, 5d and 10d [R. Pinsky, 9/3/09] (D) Change τ to τ_{101} in three places.

Page 235, 13u [R. Pinsky, 9/3/09] (D) $-\log(\varepsilon\pi_{\min})t_{\text{rel}}$ should be replaced by $\frac{2}{\varphi_\star^2}\log(\frac{1}{\varepsilon\pi_{\min}})$.

Pages 236-237. [R. Pinsky, 9/3/09] (D) Laziness is needed in the proof of Lemma 17.14, and should be added as an hypothesis. Laziness is not needed for Lemmas 17.12 and amd 17.13, which hold for general chains.

NOT FIXED **Page 240, 3d. [R. Pinsky, 9/3/09] (D)** Change martingale to supermartingale.

Page 241, line 2, Proposition 17.20 [E. Lubetzky, 12/24/09] (D) Proposition 17.20 Condition (ii) should be $Z_{t+1} - Z_t \leq B$ without any conditional expectations.

Page 241, 8d. [ELW, 7/15/2010] (E) Missing period at end of paragraph.

Page 244, 13u. [R. Pinsky, 9/3/09] (D) Change (ii) to (i).

Page 239, 5d. [R. Pinsky, 9/3/09] (D) It should be noted that $c(\beta) > 0$.

Chapter 18: The Cutoff Phenomenon

Page 250, above last display. [BFMM, 8/8/11] (D) The definition of $\tilde{X}_t$ should say that when a move below 0 is attempted, the process simply remains at 0.

Page 250, Proof of Lower Bound, Step 2. [BFMM, 8/8/11] (D) The display should be

$$\pi^{(n)}(A_h) = \frac{1 - (q/p)^h}{1 - (q/p)^{n+1}}.$$

Suitable modification to the bounds below are needed, which do not effect the argument.

Page 252, 2d. [R. Pinsky, 9/3/09] (D) Change $\leq$ to $=$.

Page 252, 3u. [R. Pinsky, 9/3/09] (D) In the statement of Proposition 18.4, reversibility should be added as an assumption.

Page 255, equation 18.24 [P. Sousi, 6/11/10] (D) $c > 0$ should be $c > 1$.

Chapter 19: Lamplighter Walks

Page 258, 9d [R. Pinsky, 9/3/09, P. Sousi 7/11/2010] (D) Change $\lesssim$ to $\leq$.

Page 258, 2u [P. Sousi, 7/11/10] (D) $F_t(w) - F_0(w)$ should be squared.

Page 259, line 1d [P. Sousi, 7/11/10] (D) There should be an absolute value around $F_t(w) - F_0(w)$.

Page 259, 19u [R. Pinsky, 9/3/09] (D) Change $\frac{1}{\log 4}$ to $\frac{1}{4 \log 4}$.

Page 260, 5d [P. Sousi, 7/11/10] (D) Replace $\mathbf{E}$ by $\mathbf{E}_x$.

Page 260, 0u [P. Sousi, 7/11/10] (D) The end of the page should have a $\blacksquare$ indicating the end of the proof.

Page 261, 10u [R. Pinsky, 9/3/09] (D) Change $t^\star_{\text{mix}}$ to $4t^\star_{\text{mix}}$.

Chapter 20: Continuous Time Chains

NOT FIXED **Page 266 Theorem 20.3 [R. Pinsky, 9/3/09] (D)** $k = k(\varepsilon)$ can be chosen so that it works simultaneously for all chains.

Page 266 Theorem 20.3 [K. Burdzy, 2/28/13] (D) Omit "Fix $\varepsilon > 0$." One should first take $k > k_0$ for an absolute constant k_0 and the statement holds for all $\varepsilon < \varepsilon_0$.

Page 268 below (20.9) [N. Berestycki, 4/09/10] (D) In equation for $u'(t)$ replace $P - I$ by $I - P$.

In the next two lines, $u(t)$ should be multiplied by γ on two occasions.

Page 269, Theorem 20.6 [N. Berestycki, 4/09/10] (D) P should be reversible.

Chapter 21: Countable State Space Chains

Page 277, Section 21.2 [R. Pinsky, 9/3/09] (D) The conductance $c(x, z_n)$ is defined to be $c(x, z)$ where z is the vertex in $V \setminus V_n$ adjacent to x.

Chapter 23: Open Problems

Page 300, Question 7 [A. Nachmias, 4/7/09] (D) The condition should be added that the graph have spectral gap bounded away from zero.

Appendix B: Introduction to Simulation

Page 320, line 3 [N.J.A. Sloane, 4/12/11] (D) The 2^{nm} should be $2^{nm/2}$.

Appendix C: Solutions to Selected Exercises

Page 328, solution to Exercise 2.3 [P. Sousi, 7/30/2011] (E) The expected time is $n^2 + n$, not n^2. (The given solution corresponds to the chain with probability 1 to go from n to $n - 1$.

Page 331, solution to Exericse 4.5 [P. Prałat, 2011] (D) $Y := (Y^{(1)}, \ldots, Y^{(n)}$ instead of $Y := (Y^{(1)}, \ldots, X^{(n)}$.

Page 331, solution to Exercise 4.5 [P. Prałat, 2011] (D) We should have $\mathbf{P}\{X_i \neq Y_i\}$ instead of $\mathbf{P}\{X_i = Y_i\}$.

Page 331, solution to Exercise 5.1 [P. Prałat, 3/24/11] (D) Since both chains need to go to x_0 we should have "at most $(1-\alpha^2)$" instead of "at most $(1-\alpha)$".

Page 332, solution to Exercise 5.4 [J. Steif, 11/2013] (D) Remove first k.

Page 332, solution to Exercise 5.4 [P. Prałat, 3/24/11] (D) On line 12d, it should read $\mathbf{P}(G_i) \leq 4^{-k}$.
Page 333, solution to Exercise 6.5 [P. Prałat, 3/24/11] (D) Replace t/t_0 by $\lfloor t/t_0 \rfloor$ in the the exponent.

Page 333, solution to Exercise 6.7 [P. Prałat, 3/24/11] (D) Here is a correct solution:

Let A be the set of vertices in one of the complete graphs making up G, excluding $v^\star$. Clearly, $\pi(A) = (n-1)/(2n-1) \leq 2^{-1}$.

Let τ be the first time the chain visits $v^\star$. For $x \in A$,

$$P^t(x, A) = \mathbf{P}_x\{X_t \in A\} \geq \mathbf{P}_x\{\tau > t\} = (1-\alpha_n)^t \tag{E-1}$$

where

$$\alpha_n = \frac{1}{2n-1} = \frac{1}{2n}\left[1 + o(1)\right].$$

The total variation distance can be bounded below:

$$\left\| P^t(x,\cdot) - \pi \right\|_{\mathrm{TV}} \geq P^t(x,A) - \pi(A) \geq (1-\alpha_n)^t - \frac{1}{2}. \tag{E-2}$$

Since

$$\log(1-\alpha_n)^t \geq t(-\alpha_n - \alpha_n^2/2)$$

and $-1/4 \geq \log(3/4)$, if $t < [4\alpha_n(1-\alpha_n/2)]^{-1}$, then

$$(1-\alpha_n)^t - \frac{1}{2} \geq \frac{1}{4}.$$

This implies that $t_{\mathrm{mix}}(1/4) \geq \frac{n}{2}\left[1 + o(1)\right]$.

Page 334, solution to Exercise 7.1, [P. Prałat, 3/24/11] (D)

- Last line: $\mathrm{Var}(N_t)$ should be $\mathrm{Var}(W_t)$.
- The inequality in line 4u should be strict, i.e., replace ≤ 0 by < 0.

Page 337, solution Exercise 8.11(a), [P. Prałat, 4/27/11] (D) t should be replaced by n.

Page 338, solution Exercise 8.11(a), [P. Prałat, 4/27/11] (D) The sequence $(U_i)_{i \geq -d+2}$ should be taken to be infinite. In the first displayed equation, replace U_{-d+1} by U_{-d+3}. We should define the j-th **order statistic** V_j among random variables $\{A_1, \ldots, A_n\}$ to be the j-th smallest among $\{A_1, \ldots, A_n\}$. The definition of $A_t^{(j)}$ should read

$$A_t^{(j)} := |\{-d+1 \leq k \leq t\} : V_{j-1} < U_k \leq V_j\}|.$$

The condition $x_j \geq 1$ for $1 \leq j \leq d$ should be added to the third display.

Page 339, [A. Frieze, 4/4/10] (D) In equation (C.18), we should have $W_2(v)$.

Page 242, solution to Exercise 12.2, [P. Sousi, 7/19/11] (D) The first sentence should say that f is an eigenfunction of $\tilde{P}$, not P.

Page 342, solution to Exercise 12.4, second equation from end, [P. Sousi, 7/19/11] (D) The bound should contain $(1-\gamma_\star)^{2t}$ instead of $(1-\gamma_\star)^2$.