391
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1) Normal distributions – PDFs & the Gaussian

2) SDOM – standard deviation of the mean

~~3) weighted averages~~

3 ~~(4)~~ Lab 2 Brownian Motion or Counting Statistics

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1) typically ~~with~~ plan for N measurements of same thing for an experiment. (then we might change a variable and go again, that's later

$x_1, x_2, x_3 \ldots x_N$

→ Our goal, then, is to develop a "best estimate" & "best width estimate" from these N ←

big picture

where are we going?

We start our statistical analysis process by assuming our distribution of measures x_i has a well-defined mean & standard deviation $\bar{x}$ & σ.

Thus the central limit thm says that M sets of these N measures would be "normal". ⟹

~~Look at stat/online & 2 results~~

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~~first we'll look at 1) increasing precision~~

Details

a) consider distribution of (10) measurements

26, 24, 26, 28, 23, 24, 25, 24, 26, 25

want to look at the distribution, so we

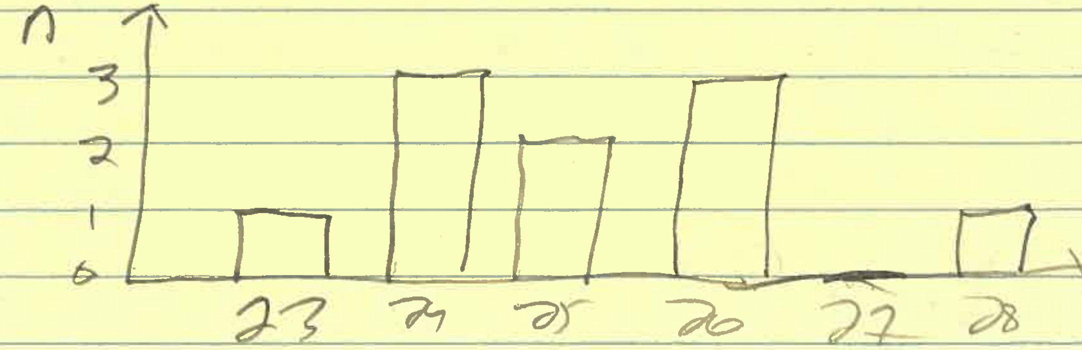
i) sort

23, 24, 24, 24, 25, 25, 26, 26, 26, (27), 28

ii count

23	24	25	26	27	28
1	3	2	3	0	1

iii plot frequency of occurrence



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iv) represent as mean

$$\bar{x} = \frac{\sum x_i}{N} = \frac{\sum_{j=1}^{m} n_j x_j}{\sum n_j} \quad (= N)$$

- j ranges over bins
- x_j value of bin
- n_j # in bin

v) could write $f_j = \frac{n_j}{N}$

$$\Rightarrow \bar{x} = \sum_j f_j x_j$$

b) aside about real world "bin widths"

normally measure real number with some accuracy say 0.1 and

so measure looks instead like the distribution of
[already ordered]

23.7, 23.8, 23.9, 23.9, 24.6, etc

0.1 0.1 0.0 0.7

look at spaces in data

pick some reasonable average of gaps as "bin widths"

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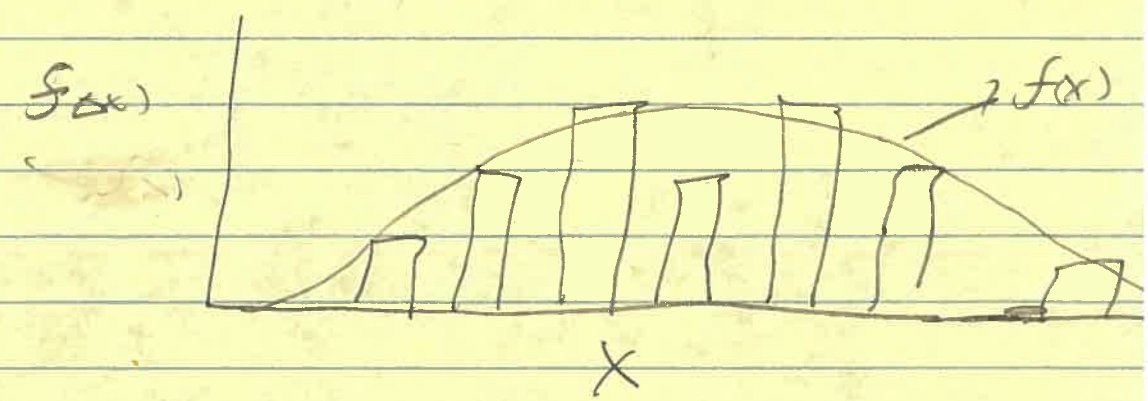
c) we're working towards a statistical description of data distribution

$f(x)$ a probability distribution fctn (it is data dependent)

so will move from



limit as $\Delta x \to 0$, $N \to \infty$ is $f(x)$

→ then $f(x)\,dx$ is the ~~fraction~~ fraction of samples between x and $x+dx$

or more ~~properly~~ formally

$$\int_a^b f(x)\,dx = \text{fraction of samples between } a + b$$

d) need to normalize s.t.

$$\int f(x)\,dx = 1$$

3 ε1

2.5
3.5

e) now, with our $f(x)$ properly normalized we can state that:

these two things define our data distribution somewhat well

i) $\bar{x} = \int_{-\infty}^{\infty} x \, f(x) \, dx$ "expectation value"

ii) $\sigma_x^2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 f(x) \, dx$ "variance"

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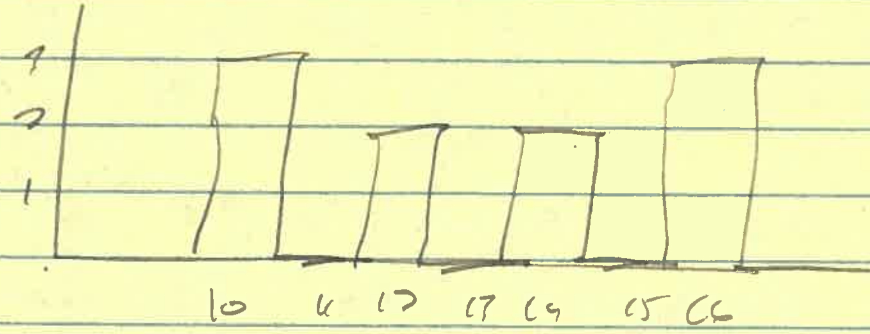
f) Central Limit Thrm

Skip

start with any distribution with <u>well-defined mean</u> and <u>variance</u>



Can set a sample size N

and take N samples (examples) of above

say $N=4$ $\Rightarrow$ $[10, 10, 14, 16] \Rightarrow \bar{x}_1 = 12.5$

keep doing this, then plot freq. dist of example

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3.6

get something that looks like a "normal distribution"

further, if we increase example size N and look at freq-distr. looks more normal

CLT says that as $N \to \infty$
freq distr. $\to$ "normal"
with well-defined mean and variance.

end
skip

note, we started with any distribution and only assumed it had a well defined mean and variance

important

it might not, itself, be "normal"

g) but what does it mean to be "normal?"

implies that $f(x) = \text{Gaussian} = e^{\frac{-x^2}{2\sigma^2}}$

or $e^{\frac{(x-\bar{x})^2}{2\sigma^2}}$

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for now we'll call σ the "width fctn"
and $\bar{x}$ the "maximally probable value"

→ would typically need to normalize this $f(x)$ s.t

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

h.) ERF

So the probability of finding ~~an~~ a sample between $\mu-\sigma$ & $\mu+\sigma$ is

$$\text{Prob}(\mu-\sigma, \mu+\sigma) = \int_{\mu-\sigma}^{\mu+\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

can let $z \equiv \frac{x-\mu}{\sigma} \Rightarrow$

$$\text{Prob} = \int_{-1}^{1} \frac{1}{2\pi} e^{-\frac{z^2}{2}}\,dz$$

called the "error fctn" or ERF

look up values in book

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just mention

21 SDOM

additional assumption
here that
orig dist
represented
by x_i
is normal

have established, given certain (reasonable) assumptions, $\bar{X} = \bar{x}$ and $\sigma_0 = \sigma$

Still working w/ N x_i's, what is "reliability" of $\bar{X} = \bar{x}$?

to consider, look at $\bar{x}_k = \frac{\sum_N x_{i,k}}{N}$ $k = 1, M$ (sets)
dist. of $\bar{x}$'s over M "sets"

we note that if $x_1, x_2 \ldots x_N$ is normally distributed then $\bar{x}$ is normally distributed (simple fcn of x_i's)

2^{nd}, each x_i has the same true value $\bar{X}$

$$\text{so } \bar{X} = \frac{\bar{X} + \bar{X} + \bar{X} \ldots}{N} = \text{true value}$$

Now look at stat-online 2 results

So measuring M sets doesn't change $\bar{X} = \bar{x}$ (at all)

Q: what is width estimate of $\bar{x}$? (add errors in $\bar{x}$ in quadrature)

$$\sigma_{\bar{x}} = \sqrt{\left(\frac{\partial \bar{x}}{\partial x_1}\sigma_{x_1}\right)^2 + \left(\frac{\partial \bar{x}}{\partial x_2}\sigma_{x_2}\right)^2 + \cdots \left(\frac{\partial \bar{x}}{\partial x_N}\sigma_{x_N}\right)^2}$$

but $\sigma_{x_1} = \sigma_{x_2} = \sigma_{x_N} = \sigma_x$

and: $\frac{\partial \bar{x}}{\partial x_1} = \frac{\partial \bar{x}}{\partial x_2} = \ldots \frac{\partial \bar{x}}{\partial x_N} = \frac{1}{N}$

$$\Rightarrow \sigma_{\bar{x}} = \sqrt{\left(\frac{1}{N}\sigma_x\right)^2 + \ldots \left(\frac{1}{N}\sigma_x\right)^2}$$

$$= \frac{\sigma_x}{\sqrt{N}} \quad \text{SDOM}$$

(what we use in reporting error)

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3) Brownian Motion

- Say we put ~5 micron spheres in deionized water at room temp

- what would we see? (nada)
  " " " " with 40x mag.? (still nada)

- what would we see if made movies with 40x digital microscope (still nada) (depends on frame rate)

- lets make frame rate 0.05 fps (ask for what the reason)

(Show movie)

we note: (ask for this)

- spheres appear to move
- "motion" is called Brownian Motion
- there may be other factors affecting their motion (gravity)

Why do this?

$$\overline{\Delta x^2} = 2 D_{\Delta x} \Delta t$$

(also $\overline{\Delta y^2} = 2 D_{\Delta y} \Delta t$

$\overline{\Delta r^2} = 4 D_{\Delta r} \Delta t$)

- first, why Δx (Δy, or)?

- next what does $\overline{\Delta x^2}$ mean?

- note that $\sigma^2_{\Delta x} = \overline{\Delta x^2} - (\overline{\Delta x})^2$

what do expect $\overline{\Delta x}$ to be for B.M?

[mention it should be ϕ, but this might be 'experiment zero' $\Rightarrow$

$(\overline{\Delta x})^2 << \overline{\Delta x^2}$

thus $\overline{\Delta x^2} \simeq \sigma^2_{\Delta x}$

- what is Δt? Could we vary it?

how is that useful?

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Aim question, what is D?
(the 'diffusion constant' first hypothesized by Einstein

with $Df = k_B T$ $T = ?$, units?

and $f = 6\pi \eta R$ (drag factor, calculable)

⟹ If we can estimate D + its error (δD) ⟸
then we can " k_B + its error

In this lab we will:

1) make movies of spheres undergoing BM

2) analyze these, first using LoggerPro

3) get LP data ⟹ python (or MC)

4) ? calc Δs, $\overline{\Delta x}^2$ vs $\overline{\Delta x^2}$ (data quality)

5) ? do statistics D_{random}

6) ? est $D + \delta D$

7) calc $k_B + \delta k_B$

8) 'Time Evolution'

ask students to complete list