Phys 341
Sp 14
Lec 18

(1)

0) Review

1) Goals of FFT

2) Properties of FFT

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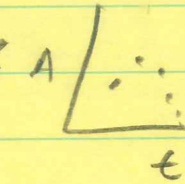
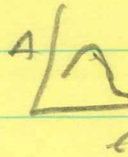
0) a.) all real data is discrete:  not 

b.) a lot of data is sampled at a constant rate (ν_s) or even spatially @ constant spacing (but some is not!)

oxford PDF

aside "Fraunhofer diffraction pattern is Fourier transform of amplitude function leaving the diffraction aperture"

c.) once ν_s & N are set, Fourier freqs are chosen

$$T_{sample} = N \Delta t = \frac{N}{\nu_s}$$

$$\Delta \nu = \frac{1}{T_{sample}} = \frac{\nu_s}{N}$$

$$\omega_f = 2\pi \Delta \nu = 2\pi \frac{\nu_s}{N}$$

ang. freqs of raw FFT are:

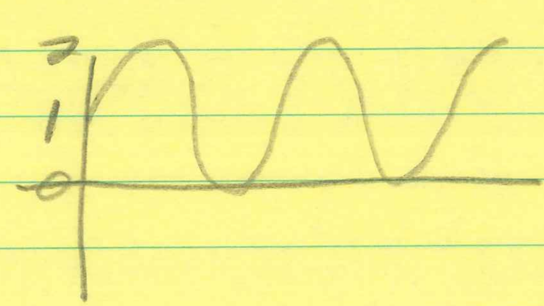
$$0,\ 1\cdot\omega_f,\ 2\cdot\omega_f,\ 3\cdot\omega_f \ \dots\ \frac{N}{2}\omega_f$$

c) where $\frac{N}{2}\omega_f$ = Nyquist ang. freq.

$$= \frac{N}{2}\frac{2\pi\nu_s}{N} = (2\pi)\frac{\nu_s}{2}$$

d) there's always a DC component ($\omega = \phi$)

check



DC comp.?

1) FFT Coefs

F.C.

freq of F.C.

phase of F.C.

$$\psi(t) = A_0 + A_1 \cos(\omega_f t + \phi_1) + \dots$$
$$A_{\frac{N}{2}} \cos\left(\tfrac{N}{2}\omega_f + \phi_{\frac{N}{2}}\right)$$

Q: why these particular functions/freqs?

A: they are orthonormal, a basis set

consider $\sum_{\text{cycle}} \cos(\omega_a t) \cdot \cos(\omega_b t)$

$\uparrow$

T_{sample}

must be $k \cdot \omega_f$

If $\omega_a \neq \omega_b$ $\quad \Sigma = \phi$

If $\omega_a = \omega_b$ $\quad \Sigma \neq 0$

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ML examples Lecture 8 on D & E

— in ML $\psi(t)$ written as:

negative freqs.

$$\psi(t) = c_0 + \sum_{n=1}^{N/2} [c_n e^{in\omega_0 t} + c_n^* e^{-in\omega_0 t}]$$

c_n's are complex except for c_0
(follow notation that $e^{in\omega_0 t} = \cos(n\omega_0 t) + i \sin(n\omega_0 t)$)

How do we estimate c_n's?

a given basis fn

$$n^{th}\ c_n \text{ from } C(\omega_n) = \Delta t \sum_{l=0}^{N-1} \psi(t_l) \cdot e^{-i2\pi \frac{nl}{N}}$$

loop for $(n = -\frac{N}{2}+1 \ldots 0, \ldots \frac{N}{2})$

$$= \Delta t\, c_n$$

So we step through each basis fctn, mult. each pt of $\psi(t)$ by corresp "pt" of basis fctn
⟹ orthonormality ensures it "picks out" that freq. component

also can do "~~inverse~~ inverse FFT" from F.C. in F.D to $\psi(t)$

$$\psi_l(t) = \sum_{n=0}^{N-1} c_n e^{+i2\pi \frac{nl}{N}}$$

note N-1 to include "negative freqs"

lth component of $\psi(t)$

typically, represent c_n's as amplitudes & phases

(bars?) $\text{amp}(c_n) = \sqrt{(\text{real}(c_n))^2 + (\text{imag}(c_n))^2}$

$\text{phase}(c_n) = \arctan \frac{\text{imag}(c_n)}{\text{real}(c_n)}$

ML (rerun F) then D shift by 1 + rerun)

ML (then do J) to show effect of N)

2) FFT properties:

Linearity if $h(t) = \alpha f(t) + \beta g(t)$

Saw this then $\hat{h}(\omega) = \alpha \hat{f}(\omega) + \beta \hat{g}(\omega)$

Translation $h(x) = f(x - x_0)$

$\Rightarrow \hat{h}(\omega) = e^{-i\omega x_0} \hat{f}(\omega)$

phase shift

Draw on board $\Rightarrow$

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Differentiation

$$h(t) = \frac{d}{dt} f(t)$$

$$\Rightarrow \hat{h}(\omega) = i\omega \hat{f}(\omega)$$

this is quite useful $\Rightarrow$ further

$$h(t) = \frac{d^n}{dt^n} f(t)$$

$$\Rightarrow \hat{h}(\omega) = (i\omega)^n \hat{f}(\omega)$$

$\Rightarrow$??

Convolution

(correlation)

If $h(t) \equiv f(t) \otimes g(t) \equiv \int_{-\infty}^{\infty} f(t_0 - t)\, g(t)\, dt$

then $\hat{h}(\omega) = \hat{f}(\omega) \cdot \hat{g}(\omega)$

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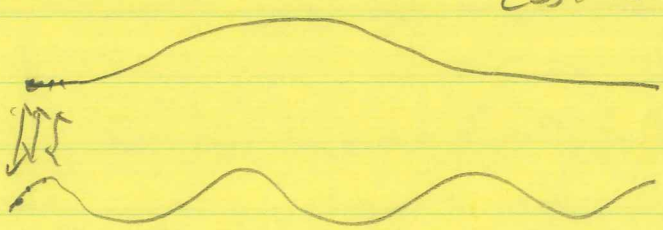
Windowing (cont'd)

So what we really get out of FFT'ing N points is a convolution of the real signal FFT and that of the window. Special windows are designed to minimize this impact

cosine² "bell"



point by point mult.

x time series

4) Lab 5

demo w/ NI DAQ 6009

talk about labs