1) weighted averages

2) Background: Bayesian MCMC theory

3) about lab write-ups

1) weighted averages

Have a separate experiment intending to measure the same thing. However, the x ± δ's are different for 2.

Goal: how to combine? or, specifically, what do we state as the "best value" and use for the 2 measurements?

We approach as before, but for the maximum likelihood maximum the JP.

\[ JP = \frac{1}{2\alpha^2} \left( \frac{X - \bar{X}}{\delta} \right)^2 + \frac{1}{2\beta^2} \left( \frac{X - \bar{X}}{\delta} \right)^2 \]

with \( x^2 = \frac{(X - \bar{X})^2}{\delta^2} + \frac{(X - \bar{X})^2}{\delta^2} \)
to maximize $J_P$, set partial derivative = 0

ie find

Equiv to minimizing $\chi^2$

\[ \chi = \frac{x_a}{2a^2} + \frac{x_b}{2b^2} \]

\[ \frac{1}{2a^2} + \frac{1}{2b^2} \]

define $w_i = \frac{1}{2a^2}$

\[ x = \frac{W_a x_a + W_b x_b}{W_a + W_b} \quad \text{weighted mean} \]

in general

\[ x = \frac{\sum w_i x_i}{\sum w_i} \]

2 Bin Theory

Consider a system of a solute (large molecule) and solvent such that:

\[ \begin{array}{c}
\text{solute} \\
\text{solvent} \\
\text{dilute} \\
\text{solvent} \\
0-5 \\
\end{array} \]

all at same initial pressure

\[ L \quad R \]

movable membrane can

\[ 0-5 \text{ solvent units} \]
What happens next?

* solute contributes to pressure on L side of barrier
  * some solute molecules bounce off
  * more

* that solute moves L to R

* net solute flow is L to R

* pressure at L = T, barrier moves R

* solute molecules can be visualized as moving in vacuum

* Brownian particles (Brownian pol(e)s) are analogous to solute particles

* \( w_{\text{kin}} = k_B T = \frac{3}{2} k_B T \)

(credit should to Paul Langevin; looks like \( k_B \))

following motion of Brownian particle / solute mol.

gap: write at \( \frac{\text{O}}{20} \)
Simplicity in}\ e^{-}\ \text{eqn of motion for 1 particle}
\text{and in 1-D radiation}\quad \frac{m}{m}\ 
\frac{d^2\mathbf{x}}{dt^2}\ =\ -6\pi n \frac{N\mathbf{r}}{\mathbf{r}}\ +\ \mathbf{X}
\text{drag force}
\text{not push from solute mol}
\text{multiplied by } X
\quad X \cdot m \frac{d^2\mathbf{x}}{dt^2}\ =\ -6\pi n N \frac{\mathbf{r}}{\mathbf{r}} + \mathbf{X}
\text{clear rewrite: correct for}
\quad m \frac{\partial}{\partial t} (x \frac{\partial x}{\partial t}) - m \left( \frac{\partial x}{\partial t} \right)^2
\quad =\ -3\pi n N \frac{\partial}{\partial x} (x^2) + \mathbf{X}
\text{average over long time relative to collision rate of solute and mol}
\quad m \frac{\partial}{\partial t} (x \frac{\partial x}{\partial t}) - m \left( \frac{\partial x}{\partial t} \right)^2\ =\ -3\pi n N \frac{\partial}{\partial x} (x^2) + \mathbf{X}
\quad \frac{1}{2} m \frac{\partial^2 x^2}{\partial t^2} + 3\pi n N \frac{\partial}{\partial x} x^2 = k_B T
\text{Einstein says } \frac{1}{2} m \left( \frac{\partial x}{\partial t} \right)^2 = \langle x^2 \rangle = \frac{3}{2} k_B T \text{ for }
To solve, set \( y = \frac{\partial}{\partial x} x^2 \)

\[ \Rightarrow \frac{dy}{dx} + \frac{6T}{m} y = \frac{2kT}{m} \]

Solution: \( y(t) = \frac{kT}{3m} + Ce^{-kt} \)

\[ \text{go to } (x,y) \text{ section} \]

\[ \Rightarrow y(t) = \frac{\partial}{\partial x} x^2 = \frac{kT}{3m} \]

or \( x^2 = \frac{kT}{3m} t \)

\[ \Rightarrow \Delta x^2 = \frac{kT}{3m} \Delta t \]

Define \( D = \frac{kT}{6m} \)

\[ \Delta x^2 = 8 \Delta x \Delta t \]

Our expression is \( D \Delta x = kT \Delta t \)

With \( T = 6 \text{ at} \)
1) continued

\[ y(t) = \frac{d}{dt} x^2 = \frac{k_B T}{3 \pi a \eta} \]

\[ x(t)^2 = \frac{k_B T}{3 \pi a \eta} t \]

\[ \Delta x^2 = \frac{k_B T}{3 \pi a \eta} \Delta t \]

with \( D = \frac{k_B T}{6 \pi a \eta} \Rightarrow \) not true for \( a \)

\[ \Delta x = \Delta x_* = 2D \Delta t \]

in lab we use \( D \eta = k_B T \Rightarrow \)

\[ F = 6\pi a \eta \]

2) notes about lab

\( d = 3.2 \) microns but Bishop may have used larger spheres 5.4um for ones marked slides

reasonable value for \( D ? \)

\( \approx 6.8 \times 10^{-1} \)

units for every thing? stated everywhere
2. Lab write-ups

   5 sections

1) abstract (write what?)
   e.g. JR

2) intro & background
   goals for lab
   also theory + assumptions
   diagram

3) results
   simple chart plot
   tables
   quality indicators
   e.g. \( \frac{(\Delta x)}{(\Delta y)} \)

4) analysis
   summary plots
   values (e.g. \( D_{av} = ?? \))
   errors

5) conclusions
   main findings
   problems
   "next time" improvements

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General notes:

- a) put histograms in report
- b) numbers have units (always)
- c) time evolution (do, e.t.?)