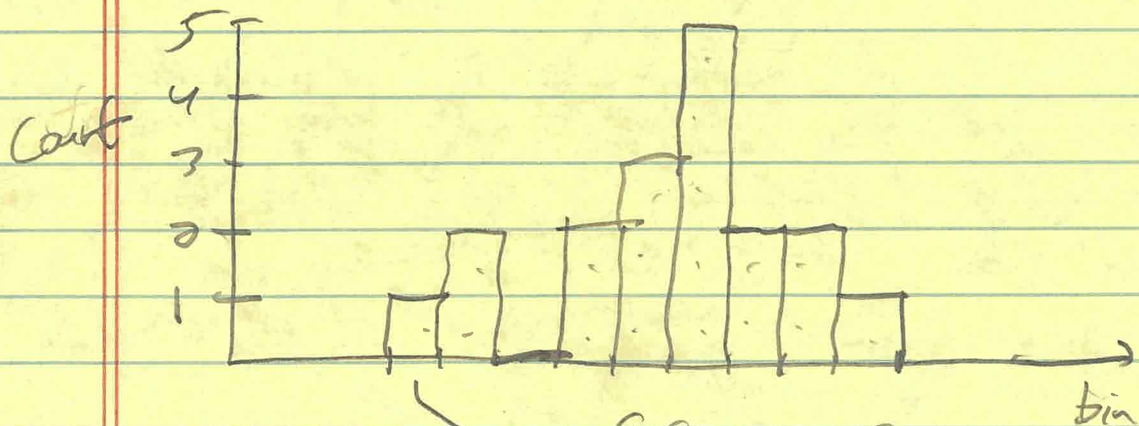
3rd Fl
L5.0

Start w/ N measurements

$$x_1, x_2, x_3 \dots x_N$$

our goal is to develop a "best estimate" $\underline{X}$

we could make N very large + make a <u>histogram</u> by counting the # of x_i's in each <u>bin</u>



a fixed range of x
Count number of observations falling within this range (1 in this case)

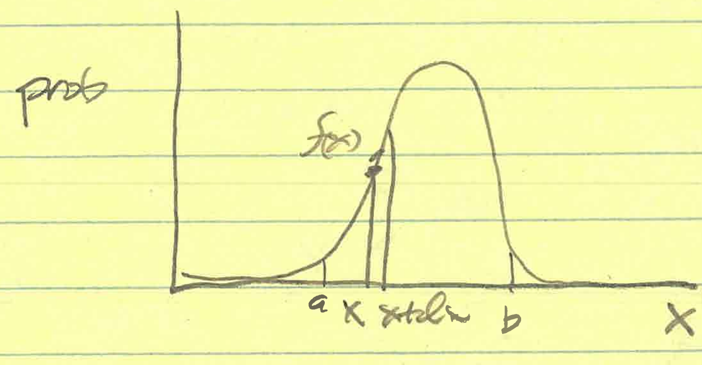
$N = 18$

If as $N \rightarrow$ large the distribution envelope of histogram, is normal*, we can use standard statistics, and assume

$$\underline{X} = \bar{x} = \frac{1}{N} \sum x_i$$

* defined next

If distribution is normal, distribution is Gaussian



probability of observ. x_i between x & $x+dx$ is $f(x)\,dx$. Between a & b is

$$\int_a^b f(x)\,dx$$

Prob $[x_i, x_i+dx]$

and $$f(x)\,dx = \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}}\,e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}}_{\text{Gaussian probability density function (PDF)}}\,dx$$

[skip to double line ~~next~~ 2 pages next]

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F16
Lk.2

1st assumption: dist. is normal

that, it's probability density fctn would be Gaussian

$$\text{Prob}[x_i, x_i + dx] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i - \bar{x})^2}{2\sigma^2}} dx$$

So now we can estimate the (joint) probability of observing our original N x's.

2nd point: (joint) probabilities are multiplicative

$$\text{Prob}_{\bar{x},\sigma}(x_1, x_2 \dots x_N) = \text{Prob}_{\bar{x},\sigma}(x_1) \cdot \text{Prob}(x_2) \cdots \text{Prob}(x_N)$$

each of these is a Gaussian

note $e^x \cdot e^y = e^{x+y}$

3rd point: JPs mult. => Σ of exponents

So joint probability =>

sum

$$\text{Prob}_{\bar{x},\sigma}(x_1, x_2 \dots x_N) \propto \frac{1}{\sigma^N} e^{\frac{-\sum(x_i - \bar{x})^2}{2\sigma^2}}$$

Q assumptions? - all σ's are same, and $\bar{x}$'s

fine, but we don't really know $\bar{x}$ + σ

4th: maximize JP

to pursue this, we'll use the "principle of maximum likelihood"

e.g. find $\bar{x}$ + σ that maximize JP

3rr
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L4.3

Q?: how to maximize JP?

4th step $\equiv$ minimize exp.

— minimize the (negative) exponent —

Q: how to do this?

$$\frac{\partial}{\partial X} \text{ of } \sum_{i=1}^{N} \frac{(x_i - X)^2}{2\sigma^2}, \text{ set to } \phi$$

(a min or max)

$$\Rightarrow \quad \sum (x_i - X) = \phi \Rightarrow$$

$$X = \frac{\sum x_i}{N} = \bar{X}$$

So the simple average is the best estimate assuming normal distribution, well-defined mean + std. dev (σ)

to find σ_b, set $\frac{\partial}{\partial \sigma}$ [JP exp.] $= \phi$ and solve

$\boxed{\text{Hw 5.26}}$

$$\Rightarrow \quad \sigma_b = \sqrt{\frac{1}{N} \sum (x_i - \bar{x})^2} = \text{our defn } \sigma_x$$

sometimes $\frac{1}{N-1}$

Phys 391
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L4.~~#3~~ B

1) weighted averages

2) background: Brownian Motion theory

3) about lab writeups

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1) weighted average

have 2 separate experiments intending to measure the same thing $x_A \pm \sigma_A$, $x_B \pm \sigma_B$

however, the $\bar{x}$ + σ's are different for 2

→ goal: how to combine? ←

or, specifically, what do we state as the "best value" and error for these 2 measurements? $\bar{X}$

do setup, work here

we approach ~~as before~~: ask for the best value $\bar{X}$ that maximizes the J.P. (maximum likelihood)

$$JP = \frac{1}{\sqrt{2\pi}\,\sigma_A} e^{-\frac{(x_A-\bar{X})^2}{2\sigma_A^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma_B} e^{-\frac{(x_B-\bar{X})^2}{2\sigma_B^2}}$$

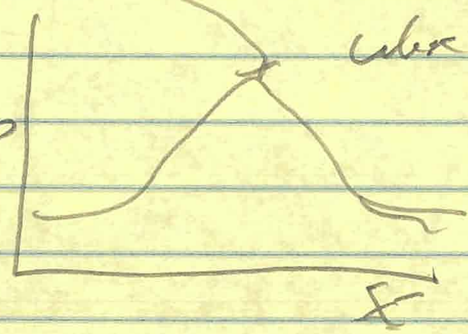
$$= \frac{1}{2\pi} \frac{1}{\sigma_A \sigma_B} e^{-\frac{\chi^2}{2}}$$

with $\chi^2 = \frac{(x_A-\bar{X})^2}{\sigma_A^2} + \frac{(x_B-\bar{X})^2}{\sigma_B^2}$

3 sq F16 L4.4

to maximize JP, set appropriate partial derivative = 0

ie find



where slope = 0

equiv to minimizing χ^2

solving $\Rightarrow$ $$\bar{X} = \frac{\frac{x_A}{\sigma_A^2} + \frac{x_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}$$

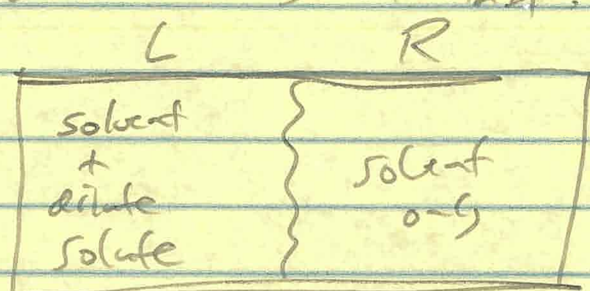
define $w_i = \frac{1}{\sigma_i^2}$ $\Rightarrow$

$$\bar{X} = \frac{w_A x_A + w_B x_B}{w_A + w_B}$$ weighted avg.

in general $$\bar{X} = \frac{\sum w_i x_i}{\sum w_i}$$

2 BM Theory

consider a system of a solute (large molecules) and solvent such that.



all at same initial pressure

moveable special membrane can pass only solvent mols

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LL4.5

what happens next?

* Solute contributes to pressure on L side of barrier
  some solute mols bounce off &

* thus more solvent moves $L \leftarrow R$ than vice versa

* net solvent flow is $L \leftarrow$

* pressure on $L \uparrow$, barrier moves $\rightarrow R$

Einstein 1905

* Solute mols can be viewed as moving in vacuum

* Brownian particles (Brown's pollen) are analogous to solute particles

* $W_{sim} = \overline{KE} = \frac{3}{2} k_B T$ for 3D

(effort shifts to Paul Langevin, looks @ $\overline{KE}$)

following motion of Brownian particle / solute mol.

goal: write out $\overline{m \frac{d^2x}{dt^2}}$ (related to $\overline{KE}$)

3rr
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L4,6

Simplify to eqn of motion for 1 particle and in 1-D

Newton's II

radius viscosity

$$m\frac{d^2x}{dt^2} = \underbrace{-6\pi a\eta\frac{dx}{dt}}_{\text{drag force}} + \underbrace{X}_{\text{net push from solvent mols}}$$

mult by x

$$x\,m\frac{d^2x}{dt^2} = -6\pi a\eta\,x\frac{dx}{dt} + xX$$

clever rewrite "corrects" for

math trick A

$$m\frac{d}{dt}\left(x\frac{dx}{dt}\right) - m\left(\frac{dx}{dt}\right)^2 = -3\pi a\eta\frac{d}{dt}(x^2) + xX$$

average over a "long time" $\Big\{$ relative to? collision rate of solvent on solute mols

trick B

$$\overline{m\frac{d}{dt}\left(x\frac{dx}{dt}\right)} - \overline{m\left(\frac{dx}{dt}\right)^2} = \overline{-3\pi a\eta\frac{d}{dt}(x^2)} + \cancel{\overline{xX}}\to 0$$

assume $(\overline{X})^2 \to 0$

(1) rewrite

$$\frac{1}{2}m\frac{d^2}{dt^2}\overline{x^2} + 3\pi a\eta\frac{d}{dt}\overline{x^2} = k_B T$$

* Einstein says $\overline{\frac{1}{2}m\left(\frac{dx}{dt}\right)^2} = KE = \frac{3}{2}k_B T$ for 3-D

$= \frac{1}{2}k_B T$ for 1-D

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L4.7

to solve, set $y \equiv \frac{d}{dt}\overline{x^2}$

$$\Rightarrow \quad \frac{dy}{dt} + \frac{6\pi a \eta}{m} y = \frac{2k_BT}{m}$$

soln $\quad y(t) = \frac{k_BT}{3\pi a \eta} + C e^{-\frac{6\pi a\eta}{m}t}$

goes to $\emptyset$ quickly, ignore

$$\Rightarrow \quad y(t) = \frac{d}{dt}\overline{x^2} = \frac{k_BT}{3\pi a \eta}$$

or $\quad \overline{x^2} = \frac{k_BT}{3\pi a\eta} t$

$$\equiv \quad \overline{\Delta x^2} = \frac{k_BT}{3\pi a \eta}\Delta t$$

define $D \equiv \frac{k_BT}{6\pi a \eta} \Rightarrow$

$$\overline{\Delta x^2} = \sigma_{\Delta x}^2 = 2D\Delta t$$

(our expression is $D\overline{f} = k_BT$
with $f \equiv 6\pi a \eta$)

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164.8

1) confirmed

$$\Rightarrow \quad y(t) = \frac{d}{dt}\overline{x^2} = \frac{k_B T}{3\pi a \eta}$$

$$\Rightarrow \quad \overline{x(t)^2} = \frac{k_B T}{3\pi a \eta} t$$

$$\equiv \quad \overline{\Delta x^2} = \frac{k_B T}{3\pi a \eta} \Delta t$$

with $D \equiv \frac{k_B T}{6\pi a \eta} \Rightarrow$ not true for Δr

$$\overline{\Delta x^2} = \sigma_{\Delta x}^2 = 2D\,\Delta t$$

in lab we use $Df = k_B T \Rightarrow$

$$f \equiv 6\pi a \eta$$

2) notes about lab

~~$d = 3.2$ microns~~ ~~but Bishop~~ may have used ~~larger spheres 5.4μ~~ for some marked ~~slides~~

a reasonable value for D?
$(\sim 6.8 \times 10^{-14})$

units for everything? stated everywhere

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F16
L4.9

2) Lab writeups

5 sections

show last → 1) abstract (write last?)
e.g. JR

2) intro & background
goals for lab
theory + assumptions } also experiment diagram

can cycle thru through "tasks" in lab writeup

3) results → Sample data plots
→ tables
→ quality indicators e.g. $\frac{(\overline{\Delta x})^2}{\overline{\Delta x^2}} \ll$
also $\overline{\Delta x^2} \simeq \overline{\Delta y^2}$

4) analysis → summative plots
→ values (e.g. $D_{\Delta x} = ??$)
→ errors

5) conclusions → main findings
→ problems
→ "next time" improvements

general notes: a) put sample histograms in report
b) numbers have units (always!)
c) time evolution (do. it?)