Understanding the CCSS through Tasks and Progressions

Dev P. Sinha

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About me

- Faculty at UO with specialty in topology.
- Undergraduate and graduate curriculum development based on variants of "active learning"; national programs and panels based on those ideas.
- Successfully adapted "active learning" techniques for courses for pre-service teachers.
- Recruited by Bill McCallum to write, comment on and edit tasks for Illustrative Mathematics Project.
- As part of that project, working with Diane Briars (past president of NCSM) and Lynn Raith as the "team mathematician" in sub-project of illustrating Standards for Mathematical Practice.
- Partnering with Lane ESD and local districts to work on PD with an eye towards curriculum improvement.

The crux of the CCSS

At first glance the CCSS looks very similar to other standards documents, such as the 2007 Oregon Standards.

Closer inspection shows a high level of care (as we'll see later when we discuss the fractions progression).

But that still misses the biggest difference, which is that CCSS-aligned mathematics is meant to be "juicier" or "meatier".

A “thin, dry” problem

6 hundreds, 5 tens, and 3 ones makes _______.

A little better

8 ones, 4 hundreds and 6 tens makes _______.

3 ones and 5 hundreds makes _______.

Now we're getting really "juicy".

12 ones and 5 tens makes ______.

13 tens and 6 ones makes ______.

3 ones, 11 tens and 4 hundreds makes ______.

Really "juicy."

Dona had cards with the numbers 0 to 9 written on them. She flipped over three of them [1,8,5]. Her teacher said:
If those three numbers are the digits in some number, what is the largest three-digit number you can make?

First Dona put the 8 in the hundreds place. Is this the right choice for the hundreds place? Explain why or why not.

Next, Dona said, "It doesnt matter what number I choose for the other places, because I put the biggest number in the hundreds place, and hundreds are bigger than tens and ones." Is she correct? Explain.

Really "juicy."

The post office packages stamps with 10 stamps in each strip and 10 strips of 10 in each sheet.

Yesterday Mike saw 4 full sheets, 7 strips, and 2 extra stamps in the drawer. He counted all the stamps and found out that there were 472 stamps in all. He said, "The number 472 matches the 4 sheets, 7 strips, and 2 stamps. Cool!"

Q1. Why did Mike's number match up with the numbers of sheets, strips, and extra stamps? Draw a picture to help explain your answer.

Today Mike found 3 extra stamps, 1 sheet, and 5 strips. He said, "Because of how things matched up yesterday, I guess there are 315 stamps total."

Q2. Find the total number of stamps. Explain why Mike's guess is incorrect. What could he have done to guess correctly?

Really “juicy”!

Lamar and Siri had some base-ten blocks.

Lamar said, "I can make 124 using 1 hundred, 2 tens, and 4 ones."

Siri said, "I can make 124 using 124 ones."

Can you find a way to make 124 using only tens and ones? Can you find a different way? Find as many ways as you can to make 124 using hundreds, tens, and ones. If you think you have found all the ways, explain how you know your list is complete.

SMP's

Instead of saying "math should be juicy," the CCSS elaborates on this concept through the *Standards for Mathematical Practice*.

If we look at "juicy" problems, we can see that students are invited to engage in the SMP's.

Meaning and consequences of "juicy"

Juicy means that a conceptual understanding is being demanded, and that some mathematical work is being done for successful completion.

Juicy does not necessarily mean "going fast" or "being abstract" or "doing exactly what they do in high-achievement countries."

Asking students to work on juicy mathematics does mean being more demanding in some sense, but students from all backgrounds should respond with a more productive disposition to such mathematics if consistently encountered at an appropriate level.

Though at first we might see some students openly struggle more, engaging juicy mathematics consistently should help close the achievement gap, especially the "hidden" part of it (which will probably not be so hidden anymore).

Challenge of curriculum development

The phrase "consistently encountered at an appropriate level" indicates the central challenge to curriculum developers.

For example, my daughter's class had a workbook with an isolated instance of the problem "Fill in $3 + 6 = \square + 4$."

This is an excellent problem, supported in its usefulness by the research literature, and aligned with CCSSM 1.OA.7.
But many/ most students would need to see such number sentences in settings where they make more immediate sense, for example in the context of pictures or other manipulatives.
Intermediate tasks such as asking "Does $2 + 3 = 4 + 1$?" would also be helpful.

CCSSM Process

- Have mathematics and pedagogy validated by teams of mathematicians and teachers.
- Base choices on both mathematics education research and on what high-performing countries do.
- Prioritize depth and thus focus, making time for juicy math.

The fractions progression

"The crown jewel" of progressions in the CCSS.

Choice I: Prioritize the number line model.

There will be more problems involving the number line itself, as well as contexts such as cutting ropes, measuring jumps and modeling the placement of landmarks.

As numbers get introduced (negative numbers, irrational numbers) they are first placed on the number line. (Ordering plays an important role here).

Even (especially?) in limited settings, juicy problems are possible: which is closer to 1 - the fraction $\frac{4}{5}$ or $\frac{5}{4}$?

The fractions progression

Choice II: Emphasize fractions as numbers themselves, not as proportions or as results of division.

Proportional relationships are introduced in sixth grade, the tail end of the progression, after multiplication of fractions has been introduced in fifth grade.
In the mathematical framework of the CCSSM, $\frac{\pi}{\sqrt{2}}$ is not a fraction.

The fractions progression

Choice III: Develop things in small steps.

The fractions progression spans from third grade, where the basic definition is given, to sixth grade, where division of fractions is treated. (Division of fractions, including its application in contextualized settings, is one of the more sophisticated topics in pre-calculus mathematics.)

Gain thorough understanding of addition of fractions with like denominators, adding other fractions through ad-hoc methods, and equivalent fractions before finally seeing the general method.

“Patterns” in the CCSSM

Choice I: Recognizing and describing patterns is an important part of mathematics, but it is more of a practice rather than a content area. In the CCSSM, recognizing and describing patterns is part of MP.7, and thus could be encountered within a variety of content areas within any grade.

For example, second-grade students can be asked to take a list of numbers from 1 to 100 in rows of 10, pick any number (not on the edge), and add its left and right neighbors as well as its “upstairs” and “downstairs” neighbors. This could be revisited during the algebra strand.

"Patterns" in the CCSSM

Choice II: Based on comparisons with high-performing countries, it was determined that a deeper grounding in arithmetic gives a stronger foundation for algebra than pattern work.

The concept of function, while significant, is less of a stumbling block for students than the fluency with the rules governing their behavior, which arise from the rules for arithmetic. For example, understanding that the equality $346 + 285 = 246 + 385$ is explained by

$$346+285 = 300+46+200+85 = 300+85+200+46 = 246+385$$

is good preparation for working with series of algebraic manipulations.

On the other hand, making patterns from colors is not demanding much, if any, mathematical work from students.

“Patterns” in the CCSSM

Choice III: “Pattern work” starts in the content standards in third grade, because students at that point are more ready (verbally as well as mathematically) to engage in such work in a mathematically well-defined way. Moreover, by having more arithmetic at play, there are more possibilities for “juicy” problems.

The problem “write the next two numbers in the sequence $4, 7, 10, \ldots$” can, mathematically speaking, be answered by putting any two numbers next. To make a well-defined problem, one should ask students to write down a rule and use that rule to give the next numbers.

The algebra strands through tasks

Seeing problems on the Illustrative Mathematics Project website is a good way to understand progressions.
Modeling will have a greater emphasis. See for example A-SSE Mixing Fertilizer (88) and A-CED Growing Coffee (611).

Note that modeling calls for more frequent engagement with inequalities (true at all grade levels) - see for example 6.EE

The algebra strands through tasks

To address the constant tension between substituting procedural understanding for conceptual, treatments emphasize the contrast and connection of conceptual and procedural.

See for example A-REI The Circle and The Line (223) and A-REI How does the solution change? (614).

Topics which are largely procedural are de-emphasized ("solving" - though it is still pretty significant, and treated much more conceptually) or cut entirely ("putting into standard form")

The algebra strands through tasks

In the middle grades, attention is placed on what it means to solve an equation, in cases where algebraic manipulation is simple or not even needed. This starts with problems for which arithmetic suffices, but then can be approached algebraically as well - see page 7 of EE Progressions, and 7.EE Discounted Books (478).

One can get quite juicy with simple linear functions or simple arithmetic. See for example 6.EE Log Ride (673) and 8.EE Fixing the Furnace (472).
In order to make modeling possible, one must take the time to emphasize the first step, namely "translating into math."
Here one sees a new focus on "reflecting on the set-up". See for example 6.EE Rectangle Perimeter 2 (461) and Page 8 of EE Progression.

The algebra strands through tasks

We see a fair amount of conceptual algebraic work at the middle grades (without the algebraic complication.) This must be grounded in experience at the elementary level.

5.OA.2 is particularly interesting in this light.
See page 32 of OA Progression, and 5.OA Words to Expressions 1 (556) and 5.OA Video Game Scores (590).

The algebra strands through tasks

Much of the formal groundwork for algebra is already being laid at grades 3 and 4.

Tape diagrams, which are a prominent part of the Singapore curricula, and variables are introduced.
See pages 27-29 of OA Progression, and 4.OA Comparing Money Raised (263) and 3.OA The Stamp Collection (13).

The algebra strands through tasks

At this point, the addition of variables does not "come from left field."

Students encounter problems in which variables are lurking behind the scences throughout their learning of arithmetic. See Tables 1 and 2 at the end of the CCSSM.

The "variable" $\square$ is introduced in first grade. See 1.OA.8, 1.OA Find the Missing Number (4), and K.OA What's Missing? (70)

The mathematics education literature has well-established the need for conceptual understanding of equality.

See for example:

http://ncisla.wceruw.org/publications/articles/AlgebraNCTM.pdf

This is addressed in the CCSSM in 1.OA.7. See for example 1.OA Valid Equalities? (466)

Different sources of juiciness: discovery

Discovery, through "experiment" and "investigation" is one of the most common approaches in reform curricula, especially at lower grades (not such much in high-school).

Rationale:

- One really understands what one figures out for one's self.
- "Give a man a fish...".
- Reasoning can be more natural.
- Discovery/ sense-making is deepest (only?) mathematical practice.

Different sources of juiciness: discovery

Example: Discovery of algorithms to add multi-digit numbers.

Challenges:

- Can take more time (note NCTM vs. CCSSM difference in mastery of multiplication facts).
- Relies heavily on teacher interaction at times and thus understanding of a variety of issues which can arise and ability to address those issues in classroom setting.
- Tension arises because of different rates of "discovery;" some students end up with more direct instruction in any case.
- Can lead to discouragement of parental involvement.

Different sources of juiciness: discovery

Suggestion: discovery with a more limited scope (closer to "patterns").
For example, multiplying $(x-2)(x+2)$ and then $(x-3)(x+3)$, etc. Tying back to numbers and rectangular regions. Working towards $(x-a)(x+a) = x^2 - a^2$.
Or for example, considering what happens when you add 9 to a two-digit number.

Different sources of juiciness: explanation

Explanation is a fundamental part of the CCSSM, addressed in MP3. In fact, many of the mathematical choices made in the CCSSM are meant to facilitate age-appropriate explanation.

Rationale

- One doesn't really understand until one can explain.
- Explanation - in the form of proof - is the currency of theoretical mathematical practice.
- Explanation embraces logic and language faculties which have ramifications across subject areas.

Different sources of juiciness: explanation

Examples:

- Showing that you can always "complete the square."
- Proofs of the Pythagorean theorem.
- Justification of procedures for multiplication and addition of fractions.

Different sources of juiciness: explanation

Challenges:

- Understanding what reasoning is age appropriate while being true to mathematical practice is a work in progress.
- Student resistance when they "just know"
- Can devolve into recounting of procedure.

Different sources of juiciness: explanation

Suggestion: set up problems to be clearer about basis for reasoning.

To set up a straw man, consider a task like "add 35 and 37, and explain how you know your answer is correct."
It could elicit "... then when I had twelve ones I made some of them into a ten and...".
But it could also elicit "... and then I carried the one, and then I added that to the three and the three, and then..."
This poses a significant teaching dilemma.

Better "add 35 and 37 by hand, and then use base-ten blocks to explain your process."

Different sources of juiciness: novelty and challenge

Novel and challenging problems are ultimately what we want our children to be able tackle, in many venues.

Rationale:

- Solving problems is the heart of mathematics, and a true problem requires some aspects of novelty and challenge (compare with MP1).
- By solving harder problems, one reflects on the basics, and thus understands them. (A popular approach to "fill in" direct instruction, especially postsecondary.)
- Math is an ideal venue to work on problem-solving habits. Scientific problem resolution for example cannot be really engaged at younger grades (because of the need for mathematics(, measurement, etc), while mathematical problem resolution can.
- Students can get bored (sometimes "checking out") if they are not challenged.

Different sources of juiciness: novelty and challenge

Example: "starred problems" in traditional textbooks. Math competitions.

Challenges:

- Problems must be approachable, or else students can tune out.
- The challenging part of the problem provides mental exercise, but may not serve any content goals.
- Problem-solving per se is awkward to "teach."
- Too often only the fastest (not even necessarily the "best") students get to work on the "challenge problems."

Different sources of juiciness: novelty and challenge

Suggestion: emphasize novelty in the course of introduction of topics, and difficulty sometimes coming from the lack of standard tools.

Examples could be seen throughout potential CCSSM curriculum, such as the "Which is closer to 1" task, or the potential to use ad-hoc methods to find lengths in triangles which soon are "easily" resolved through sine-cosine laws.

Different sources of juiciness: context and connection

Incorporate context, the need for interpretation, connections with different subject areas.
Rationale:

- Almost all mathematics which people actually do is "applied." Mathematical and statistical modeling continue to expand their reach.
- Interesting contexts can draw students in.
- Modeling, especially if it is "real world", can lead students to embrace levels of complexity which they are unlikely to embrace when imposed.

Different sources of juiciness: context and connection

Non-example: most “word problems.” Example: salty water task.

Challenges:

- Tension between realism and feasibility, and general difficulty in construction.
- The time taken up by context can “crowd out” mathematical time(!).

Different sources of juiciness: context and connection

Suggestions:

Carefully scrutinize “modeling” tasks before working with them.

Look for opportunities to connect with other subjects and thus avail yourself to some of their class time, in addition to your own, for the practice of mathematical modeling.

Two last take-aways

I think a varied diet of juiciness is important.

Being comfortable with a range of ways in which students can more deeply engage in mathematical practice will better meet the demands of the students' learning styles and the underlying mathematics.

Teachers comfortable with all of these will be better able to blend them, and play to their own strengths.

Two last take-aways

Juiciness won't mean as much "additional" time as you might think, since class time must be used for conceptual work in any case. The question is whether the conceptual work is built-in coherently or must be "back-filled" or sometimes remediated.

Scenario one: students have taken the time to do juicy tasks to understand basic properties of addition and equality ("Does $3+5+2=2+3+5$? Give two different explanations.") and place value ("Six tens and twelve ones makes"). When it comes time to learn the standard algorithm for adding two-digit numbers, they are well-equipped.

Scenario two: students must be reminded the steps and have the meaning of the algorithm explained repeatedly as they struggle to perform it relying on memory.

Lane PD Model: K-12/ higher-ed partnership

In Lane County we have put together some professional development to help teachers better understand juicy problems and the CCSSM.

We have also been including some higher-ed participants, namely undergraduate and graduate students, instructors, and faculty from the UO Math Department.

This is in line with the model adopted nationally in CCSSM and elaborative development. Teachers and mathematicians each have some insight and abilities to share on the topic of age-appropriate mathematical reasoning.

Resources

Illustrative Mathematics Project:
http://illustrativemathematics.org/

Common Core Progressions:
http://math.arizona.edu/∼ime/progressions/

Common Core Tools blog:
http://commoncoretools.me/

Description of further focus within the CCSSM:
http://www.caboces.org/sites/default/files/A
%20Common%20Core/ContentPriorities-2011-05-31-1212.pdf

1. Take the number 23 and write it, then double it, then double that, etc. a total of six times.
2. Take the first number on your list (namely 23), the fourth number and the sixth number, and add them together.
3. Compare what you got in the previous step with 23×41.
4. Compare the fourth number on the list to 23×8, and the sixth number to 23×32.
5. Explain what is going on in the previous calculations. Using your list, can you quickly calculate 23×18?

Partial solution:
The list can be written

$$23 \times 1, \quad 23 \times 2, \quad 23 \times 4 \quad 23 \times 8, \quad 23 \times 16 \quad 23 \times 32.$$

Adding the first, fourth and six gives

$$23 \times 1 + 23 \times 4 + 23 \times 16 = 23 \times (1 + 4 + 16) = 23 \times 21.$$

This is a prime example of "algebraic thinking."