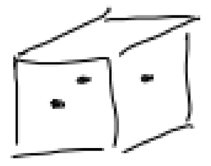
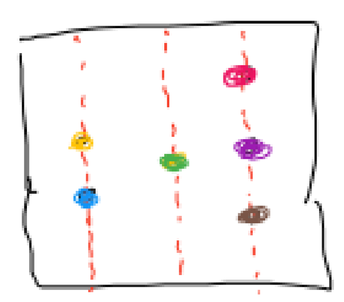
Wonderful coincidence:

$B^d C_2 =$  . Let F_n be subspace of $\le n$ points are labeled by $g \ne e \in C_2$.

$F_n B^d C_2 / F_{n-1} \underset{\text{homeo}}{\cong} U\mathrm{Conf}_n(\mathbb{R}^d)^+$ (unlabeled; 1-pt compactification)

Iterated bar cell structure on former gives rise to a cell structure $U\mathrm{Conf}_n(\mathbb{R}^d)^+$.

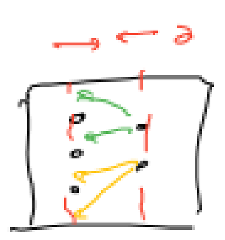
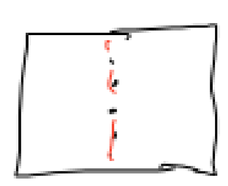
$| 1g1g1 | 1g1 | 1g1g1g1 | \longleftrightarrow$  B^2C_2

1 2 2 1 1 $\longleftrightarrow$ 1 0 0 1 1

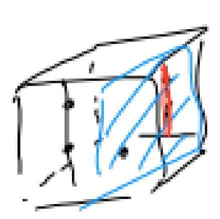
$2-x$ # shared coords

See also work of
- Fuchs
- Milgram
- Joyal (Ayala-Hepworth) Θ_n
- Salvetti
- Vassiliev

Cells easy, boundaries involved.

 $\xrightarrow{\partial} \pm$  $\pm \dots - 2$ 

$[1,1,0,1]$ (ten terms) $[1,1,1,1]$



$\partial[2,0,1,2] = -3[2,0,2,2] - [2,1,1,2]$

$\partial[0,2,1,2] = -6[0,2,2,2] - [1,2,1,2] + [2,1,1,2] - [2,1,2,1]$

Question/Projects: How does the Hopf ring structure on $\bigoplus_d H_* B^d C_2$ manifest in cell str?

Revisit Cartan's calculations.

Duality

$$\begin{array}{ccc} W^{d-n} & & (W^{d-n})^+ \\ \downarrow & \rightsquigarrow & \downarrow \\ M^d & & (M^d)^+ \end{array}$$

cohom class τ_W; proper; homology class

$H^n(M) \longrightarrow H_{d-n}(M)$

Applied here,

$$\bigoplus_{n,d,i} H_i\, F_n/F_{n-1}\, B^d C_2 \cong \bigoplus_{n,d,i} H_i(U\mathrm{Conf}_n(\mathbb{R}^d)^+)$$

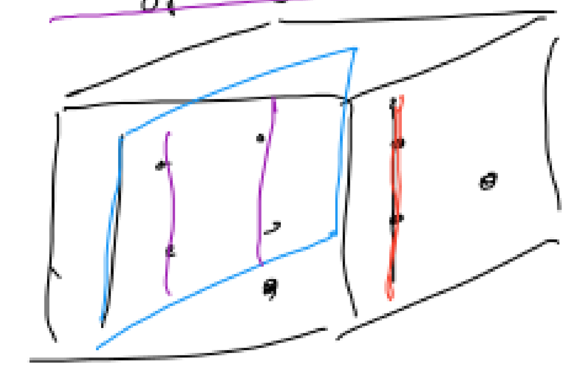
$\Vert$ mod-two F_n splits $\qquad \Vert$

$$\bigoplus_{i,d} H_i(B^d C_2) \cong H^{nd-i}\, U\mathrm{Conf}_n(\mathbb{R}^d)$$

as $d \to \infty$ get BS_n — $\exists$ homological stability & sensible limit here

On right-hand side, submflds defined by "sharing coordinates" to obtain cocycles (& thus cycles on LHS)

EX $\;\subseteq \mathrm{Conf}_8(\mathbb{R}^4)$: Two points sharing 3 coords & 4 pts sharing 1 coord which then decompose as 2 grps of 2 which share an add'l coord.

 $\mathbb{R}^4$

Corresponding Fox-Neuwirth cycle:

$[3,0,2,1,2,0,0] + [0,2,1,2,0,3,0]$

5 $\quad$ 4+2+3+2 $\quad$ 4 4

"Block symmetrization"

$H_{24}(B^4 C_2) \longleftrightarrow H^8(U\mathrm{Conf}_8(\mathbb{R}^4))$

Question/observation

$H^5(B^2C_2) \longleftrightarrow H_3(U\mathrm{Conf}_4(\mathbb{R}^2))$

$Sq^2 Sq^1 z \longleftrightarrow$