Background idea: understand differential topology through (alg top) of Configuration space functor.

Ex • Longoni - Salvatore $\exists\ L_{p,m} \simeq L_{p,n}$ but $Conf_2(L_{p,m}) \simeq Conf_2(L_{p,n})$

$\Downarrow$

$L_{p,m} \not\cong L_{p,n}$

• Kontsevich; Bott - Taubes, Volic

$f: S^1 \to \mathbb{R}^3$ knot

$Conf_n[f]$ encodes finite-type/quantum knot invts.

first ex $L: S^1 \sqcup S^1 \to \mathbb{R}^3$

$Conf_{1,1} L : S^1 \times S^1 \to Conf_2(\mathbb{R}^3)$.

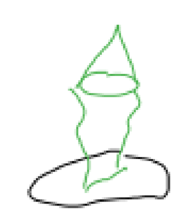
degree = Gauß $\int$.

• Watanabe: $B\,Diff\, S^4 \not\simeq B\,SO(5)$ through conf. space integrals.

Goodwillie "cutting" away handles relates embedding questions to such for disjoint unions of balls.

Cubical diagrams & Blakers Massey.

$f: X \to Y$ (ho) cofib f = $Y \cup (X \times I) / {x \times 0 \sim f(x) \atop x \times 1 \sim x' \times 1}$



" H_* - difference"

LES: $\to H_*(X) \xrightarrow{f} H_*(Y) \to H_*(Cof\, f) \to H_{*-1}(X) \to \cdots$

(ho) fib f = $\{x \in X,\ \gamma: I \to Y$ from $f(x)$ to $*\}$

" π_* - difference"

LES $\to \pi_*(fib\, f) \to \pi_*(X) \to \pi_*(Y) \to \cdots$

Defn We say $f: X \to Y$ is k-connected if fib f is (k-1) ctd space.

Cubical diagrams. $\underline{n} = \{1, \ldots, n\}$, $2^{\underline{n}}$ = subsets of $\underline{n}$

$\hookrightarrow$ functor $2^{\underline{n}} \to \mathcal{C}$ (= Top).

$2^{\underline{0}}$ = objects $\quad 2^{\underline{1}}$ = morphisms $\quad 2^{\underline{2}}$ squares.

$\emptyset \subseteq \{1\}$

Defn Total cofiber of a $\underline{n}$ cubical diagram is cofiber of induced map of $\underline{n-1}$ cubes which $\underline{n}$ cube yields

Ex
$$\begin{array}{ccccc} X_\emptyset & \to & X_2 & \to & Cof\, f_{\emptyset \subseteq 2} \\ \downarrow & & \downarrow & & \vdots \\ X_1 & \to & X_{12} & \to & Cof\, f_{1 \subseteq 12} \\ & & & & \downarrow \end{array}$$

Tot cof = cof here. = $X_\emptyset \times I^2 \cup X_2 \times I \cup X_1 \times I \cup X_{12} / \sim$

Defn A square is Co-Cartesian if Total cof $\simeq *$.

$\Rightarrow$ M-V sequence in H_*.

A square is k- co-cartesian if Total cof is k-ctd.

(M-V in a range)

Defn Total fiber is... (fibers of fibers...)

.. M-V seq in π_*.

Thm (Blakers-Massey)
$$\begin{array}{ccc} X_\emptyset & \xrightarrow{k_1\text{-ctd}} & X_1 \\ {\scriptstyle k_2}\downarrow & & \downarrow \\ X_2 & \longrightarrow & X_{12} \end{array} \quad \text{(ho) Co-cart.}$$

$\Rightarrow$ it is $(k_1 + k_2 - 1)$ cartesian.

Ex
$$\begin{array}{ccc} S^n & \xrightarrow{n\text{-ctd}} & * \\ {\scriptstyle n}\downarrow & & \downarrow \\ * & \longrightarrow & S^{n+1} \end{array}$$
. $2n-1$ Cartesian $\Rightarrow$ $S^n \to \Omega S^{n+1}$ $2n-1$ ctd.

Freudenthal.

Pf (part of special case - see Calculus II)

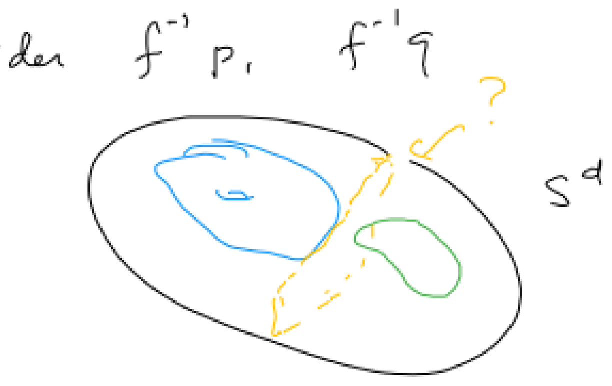
$$\begin{array}{ccc} X & \hookrightarrow & X \cup_h D^n \\ \downarrow & & \downarrow \\ X \cup_g D^m & \longrightarrow & X \cup_h D^n \cup_g D^m \end{array} \quad h: \partial D^n \to X$$

Let $f: S^d \to X \cup D^n \cup D^m$. Want to show f is sum of maps to $X \cup D^n$ & $X \cup D^m$.

Make $f \pitchfork$ $p = 0 \in D^n \subseteq X \cup D^n \cup D^m$

$q = 0 \in D^m$

Consider $f^{-1}p$, $f^{-1}q$



Can separate $f^{-1}(p)$ & $f^{-1}(q)$ if

$\dim f^{-1}(p) + \dim f^{-1}(q) < d-1$

"

$d - n + d - m < d - 1$

$d < m + n - 1$