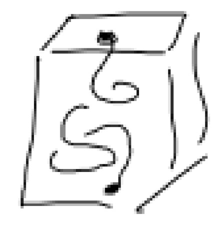
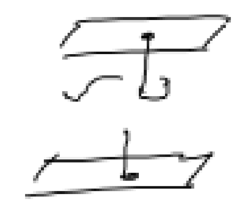
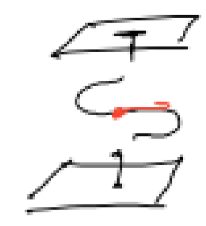
Consider $\mathrm{Emb}(\mathbb{I}, \mathbb{I}^d)$  fixed endpts & tgt vectors.

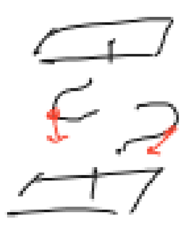
Fix $J_i \subseteq \mathbb{I}$, $i \in \underline{n}$ 

$S \subseteq \underline{n}$

$\mathrm{Emb}_S = \mathrm{Emb}(\mathbb{I} - \bigcup_{i \in S} J_i, \mathbb{I}^d)$

$\# S = 1$  $\simeq *$

$\# S = 2$  $\simeq \mathbb{I}^d \times S^{d-1}$

$\# S = 3$  $\simeq \mathrm{Conf}_2 \mathbb{I}^d \times (S^{d-1})^2 = \mathrm{Conf}_2' \mathbb{I}^d$

These form a cubical diagram.

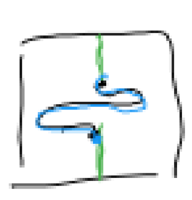
Claim: This cubical diagram is $n(d-3) - \varepsilon$ Cartesian

$\overset{d>3}{\Rightarrow}$ can recover π_* of $\mathrm{Emb}(\mathbb{I}, \mathbb{I}^d)$ from SS built from $\pi_*(\mathrm{Conf}_n'(\mathbb{I}^d))$

for $d > 3$, $\mathrm{Emb}(\mathbb{I}, \mathbb{I}^d)$ ctd but has interesting & impactful (higher) homotopy gps.

Pf (1) Take vertical fibers

image of $\mathbb{I} \setminus J_2$ under std. emb.

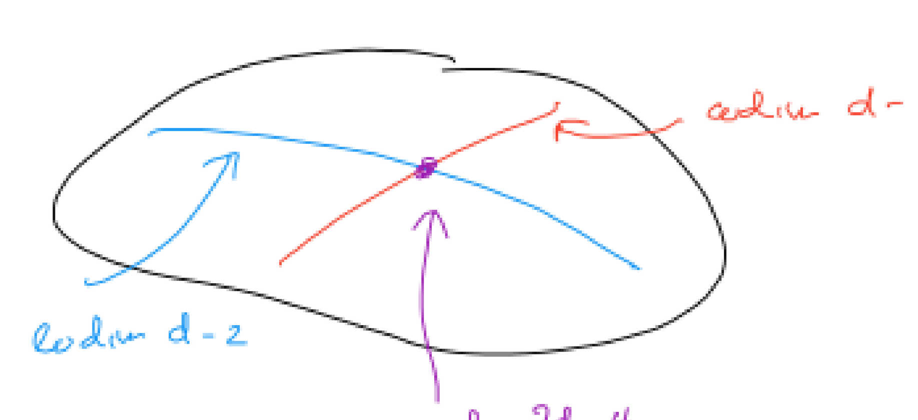


$$
\begin{array}{lll}
\mathrm{fib} = \mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_2) \longrightarrow \mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{1,2}) & & \\
\qquad \searrow \mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{2,3}) \longrightarrow \mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{1,2,3}) & & \\
\text{Emb space } \mathbb{I} \hookrightarrow \mathbb{I}^d = E_\emptyset \longrightarrow E_1 & & \\
\qquad \searrow E_3 \longrightarrow E_{13} & & \\
\text{Emb } \mathbb{I} \setminus J_2 = E_2 \longrightarrow E_{12} & & \\
\qquad \searrow E_{23} \longrightarrow E_{123} & &
\end{array}
$$

(2) Compare $\underbrace{\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{1,2,3})}_{\text{Can hit either } J_1 \text{ or } J_3} \supseteq \underbrace{\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{1,2})}_{\text{Can hit in } J_1 \text{ forbidden in } J_3} \cup \underbrace{\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{2,3})}_{\text{can hit in } J_1 \text{ can't in } J_3} \supseteq \underbrace{\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_2)}_{\text{can't hit either}}$

Fact $W \subseteq M$ codim n

$\Rightarrow$ $M \setminus W \hookrightarrow M$ is $n-1$ ctd.



$\Rightarrow$ the inclusion of union is $2d-5$ ctd.

$\Rightarrow$ through dimension $2d-5$ can replace the terminal space in the square of fibers to have a co-cartesian square

(3) Apply Blakers-Massey to

$$
\begin{array}{ccc}
\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_2) & \xrightarrow{d-3 \text{ ctd}} & \mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{2,3}) \\
\Big\downarrow {\scriptstyle d-3 \text{ ctd}} & & \Big\downarrow \\
\mathrm{Emb}(J_2, \mathbb{I}^d \setminus I_{1,2}) & \longrightarrow & \text{union}
\end{array}
$$

$\Rightarrow$ $\underline{2(d-3) - 1} = 2d - 7$ Cartesian.

Recap In general $n(d-3) - \varepsilon$ ctd

$\Rightarrow$ can recover $\pi_* \mathrm{Emb}(\mathbb{I}, \mathbb{I}^d)$ from $\pi_* \mathrm{Conf}_n'(\mathbb{I}^d)$ (better as $n \to \infty$)