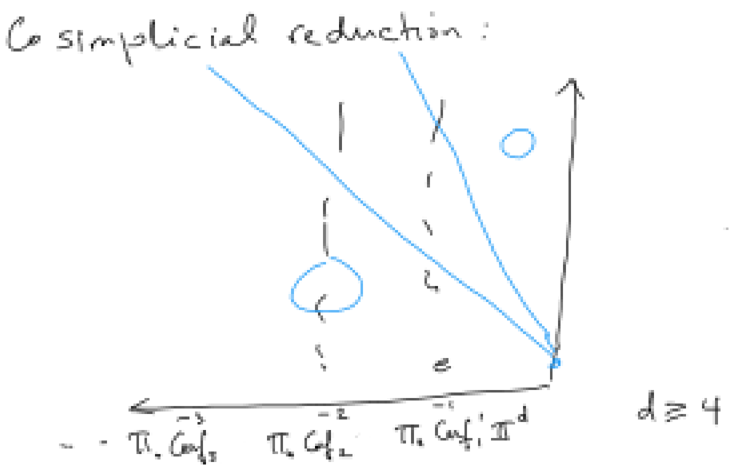
Automatically: $\exists$ SS starting w/ $\bigoplus_{i} \bigoplus_{\binom{n}{i}} \pi_* \mathrm{Conf}_i(\mathbb{I}^d) \Rightarrow \pi_* \mathrm{Emb}(\mathbb{I}, \mathbb{I}^d)$

through a range which increases w/ n.

Cosimplicial reduction:



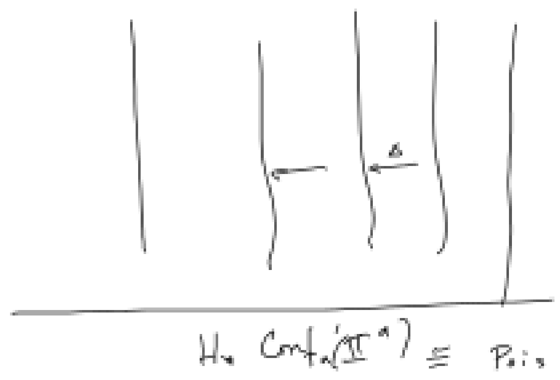
Recall: $\pi_i(\mathbb{Z}X) \cong H_i(X)$.
Dold-Thom

$\mathrm{Emb} \xrightarrow{\simeq \text{range}} \mathrm{holim}\ \mathrm{Conf}_{\bar{i}}$ ....

gen. EMSS valid $d > 3$.

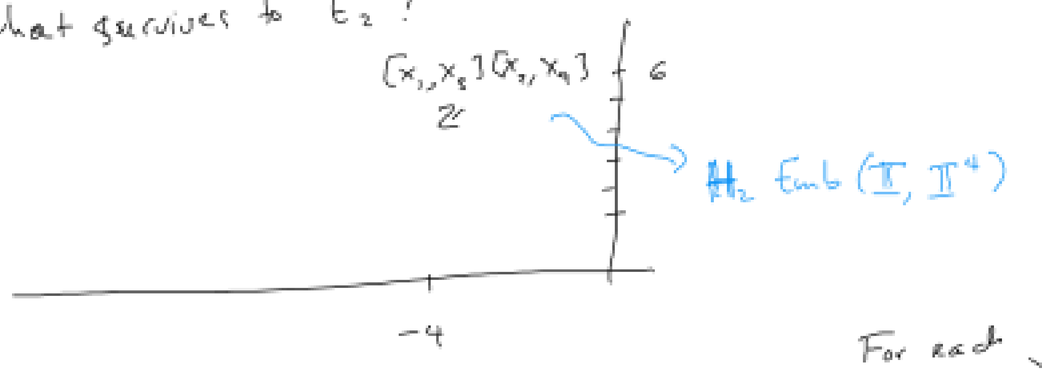
$\mathbb{Z}\,\mathrm{Emb} \xrightarrow{\simeq \text{rng}} \mathbb{Z}(\mathrm{holim}\ \mathrm{Conf}_i) \longrightarrow \mathrm{holim}(\mathbb{Z}\,\mathrm{Conf}_i)$

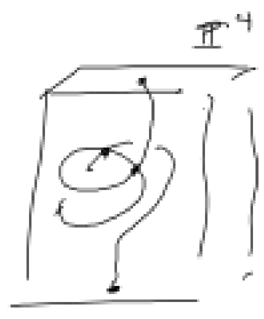
SS



$H_* \mathrm{Conf}_n(\mathbb{I}^d) \cong$ Pois (forests)
(brackets)

$d = 4$
what survives to E_2?





For each $V^2 \leq \mathbb{R}^4$, $(V^2)^\perp$ 2-D $\Rightarrow S^1 \times S^1 \longrightarrow \mathrm{Emb}(\mathbb{I}, \mathbb{I}^4)$

$v, w \longrightarrow$ "resolve 1st dble pt in v dir.
2nd " " " w dir."

$[S^1 \times S^1] \in H_2\ \mathrm{Emb}(\mathbb{I}, \mathbb{I}^4)$

These H_*-SS's are known to collapse/$\mathbb{Q}$ by formality results.

Rational homotopy Spaces $\rightsquigarrow$ DGCA's

Diagrams $\rightsquigarrow$ "biComplexes"

$\mathrm{Conf}_n(\mathbb{R}^d)$ and these "diagonal" maps (& operad maps, ..)
are formal.

Defn Strong formality $H^*(X) \xrightarrow{\simeq} C^*_{dR}(X)$ ($\Rightarrow$ no Massey products)
$d = 0$.

Consider $\mathrm{Conf}_n(\mathbb{C})$. ($\mathbb{C} = \mathbb{R}^2$) This is strongly formal.

$H^* \cong \mathbb{Z}[a_{ij}]/\text{Arnold} \longrightarrow C^*_{\mathbb{C}\,dR}\ \mathrm{Conf}_n(\mathbb{C})$

$z_1 \dots z_n$ distinct

$a_{ij} \in H^1$

$i \to j \to l,\ j \to k \longleftrightarrow a_{ij}\, a_{jk}\, a_{jl}$.

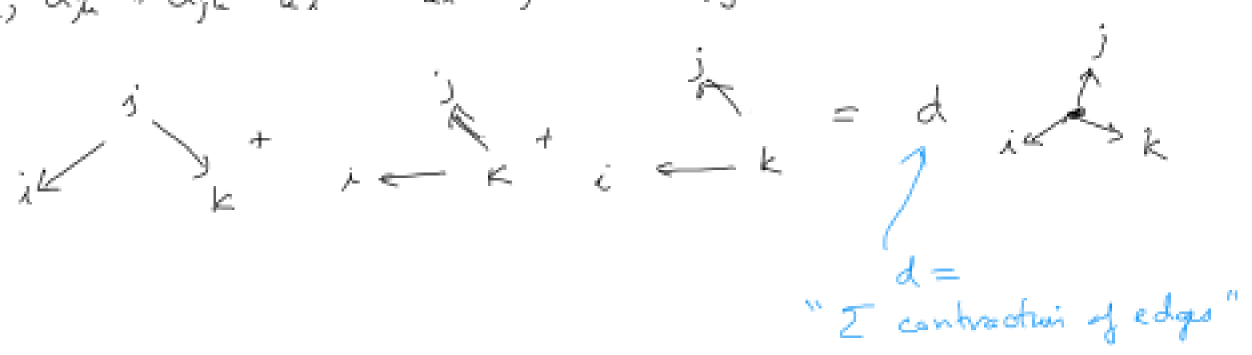
$a_{ij} \longrightarrow d\log(z_j - z_i) = \dfrac{dz_j - dz_i}{z_j - z_i}$

Arnold identity holds at cochain level!

$\Rightarrow$ strong formality

In general, Kontsevich (Bott-Taubes &..)

$a_{ij}\, a_{jk} + a_{jk}\, a_{ki} + a_{ki}\, a_{ij} = d\,\theta_{ijk}$



graphs $\rightsquigarrow$ forms
using integrals arising in QFT.