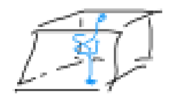
Knot: $\mathbb{I} \xrightarrow{f} \mathbb{I}^3$ 

$\rightsquigarrow$ induced map on $\mathrm{Conf}_n\langle\ \rangle$

$$\begin{array}{ccc} \mathrm{Conf}_n\langle \mathbb{I} \rangle & \longrightarrow & \mathrm{Conf}_n\langle \mathbb{I}^3 \rangle \\ \cong & g_n f,\ ev_n f,\ \mathrm{Conf}_n\langle f\rangle & \cap \\ \Delta^n & & \mathrm{Conf}_n(\mathbb{I}^3) \end{array}$$

<u>Question</u>: what of f does $g_n f$ "see"?

<u>Conjecture</u>: $\pi_0(ev_{n+1} f)$ serves as a universal Vassiliev invariant over $\mathbb{Z}$ of degree (type) n.

$ev_n(f) \in \operatorname{holim} \mathrm{Conf}_n'\langle \mathbb{I}^3 \rangle$

$=: P_n \mathrm{Emb}(\mathbb{I}, \mathbb{I}^3)$ (or just P_n)

$\subseteq \mathrm{Maps}_{\mathcal{F}}(\Delta^n, \mathrm{Conf}_n'\langle \mathbb{I}^3 \rangle)$

$\mathcal{F}$: each face of Δ^n maps to corresp. subspace of Conf.

$t_i = t_{i+1} \rightsquigarrow$ "$x_i = x_{i+1}$"

Budney, Conant, Scannell, S–.

$\pi_0(P_3) \cong \mathbb{Z}$, identified w/ $\pi_2(\mathrm{Conf}_3^{\mathrm{fib}}(\mathbb{I}^3)) \cong \mathbb{Z}$.

P_3 Emb$(\mathbb{I}, \mathbb{I}^3)$; $\mathrm{Conf}_3(\mathbb{I}^3) \to (\mathrm{Conf}_2(\mathbb{I}^3))^3$

$ev_3: \pi_0 K \longrightarrow \pi_0 P_3$ (K = Emb$(\mathbb{I}, \mathbb{I}^3)$)

is the integervalued type 2 invariant.

$\xrightarrow{ev_3 f} \mathrm{Conf}_3\langle \mathbb{I}^3 \rangle$

$\supseteq \mathrm{Col}_1 = \{x_1, x_2, x_3$ collinear, x_1 in middle$\}$; $\supseteq \mathrm{Col}_3$, x_3 in middle

x_j x_i x_k

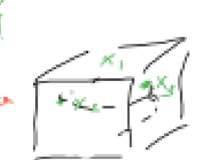
$\partial(ev_3 f^{-1}(\mathrm{Col}_1))$?

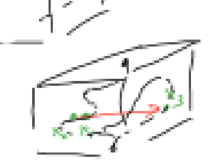
~~$0 = t_0 = t_1$~~ no preimage on this face

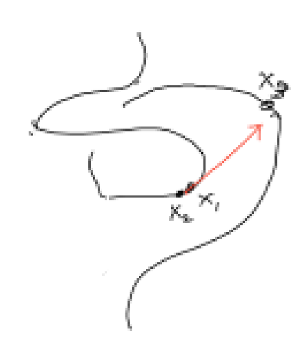
$t_1 = t_2$ ✓

~~$t_2 = t_3$~~

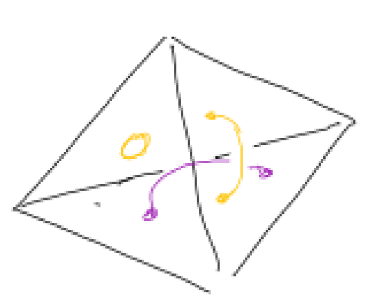
$t_3 = t_4 = 1$ ✓







$\partial(ev_3 f^{-1}(\mathrm{Col}_3))$ lies in $t_0 = t_1$ & $t_2 = t_3$ faces but not the others.



f up to isotopy class $\rightsquigarrow$ $ev_3 f$ up to homotopy

$\rightsquigarrow \mathrm{Lk}(ev_3 f^{-1}\mathrm{Col}_1, ev_3 f^{-1}\mathrm{Col}_3)$

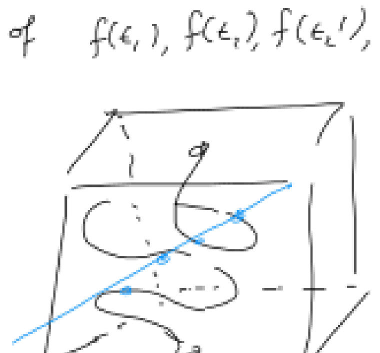
Calculate Lk by projection onto t_1, t_3 words

So collinearity $f(t_1), t_2, t_3$

another t_1, t_2', t_3

$\Rightarrow$ collinearity of $f(t_1), f(t_2), f(t_2'), f(t_3)$

|| alternating quadrisecant

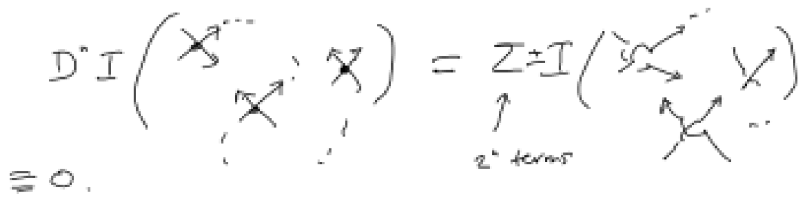


No further similar examples of knot invts obtained through analysis of $\Delta^n \xrightarrow{ev_n f} \mathrm{Conf}_n\langle \mathbb{I}^3 \rangle$

Jones polynomial

$\rightsquigarrow$ Witten / Bar Natan

Vassiliev / Birman-Lin



Type n := $D^{n+1} I \equiv 0$.

Kontsevich / Bott-Taubes (... Cattaneo .. Willwacher) give integral realizations

Habiro / Goussarov $\rightsquigarrow$ an equiv reln. on knots.

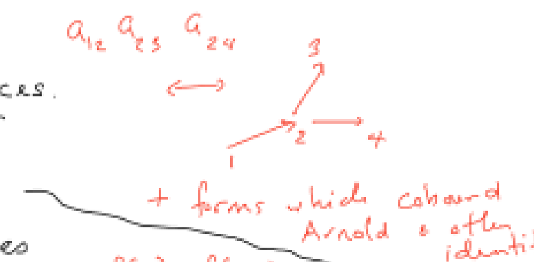
Conant - Teichner (... Kosanovic) $\rightsquigarrow$ a different equiv reln.

What role(s) do graphs play?

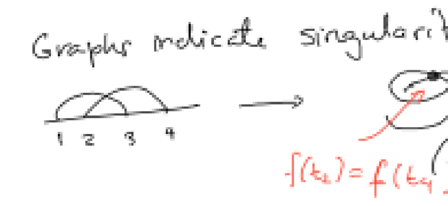
Witten: Feynman diagrams.

Kontsevich: Differential forms on configuration spaces.

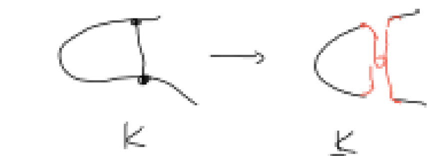
Bott: Role these play in $\mathbb{Q}$-homotopy of ↑.



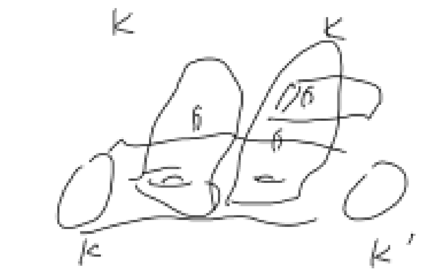
Vassiliev / Birman-Lin: Graphs indicate singularities



Habiro - Goussarov: 'moves' 'surgeries'

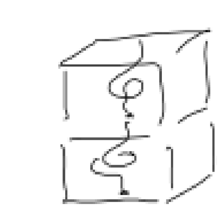


Conant - Teichner: gropes



Proving the main conjecture will almost certainly lead to a new interpretation of graphs in this context.

<u>Progress</u> (Budney, Conant, Koytcheff, S–):



– $\pi_0(ev_{n+1})$ is an abelian gp homomorphism.

– It is type n. (invt. under Habiro moves)

– Spectral sequence is one of ab gps for π_0 & π_1.

– The E_2 terms in degrees $(-n+1, n+1)$, which are quotients of $\pi_{n+1}(\mathrm{Conf}_{n+1}(\mathbb{I}^3))$, are $\cong$ to the conjectured form for universal type n invt. over $\mathbb{Z}$.

– Quotients of free $\mathbb{Z}$-modules $\Rightarrow$ Hopf invts should detect them.

<u>Progress</u>: Kosanovic: $\pi_0 K \longrightarrow \pi_0 P_{n+1}$ is surjective. (2006 Volic conj/$\mathbb{Q}$ but using Bott-Taubes. related.)

2020 Boavida de Brito - Horel: mod-p SS collapses in degrees less than p.

$\Rightarrow$ (mod-p) Conjecture in those degrees

But techniques: Grothendieck-Teichmuller actions

No new explicit knot invts.