CLASSIFYING SPACES

①

homotopy type

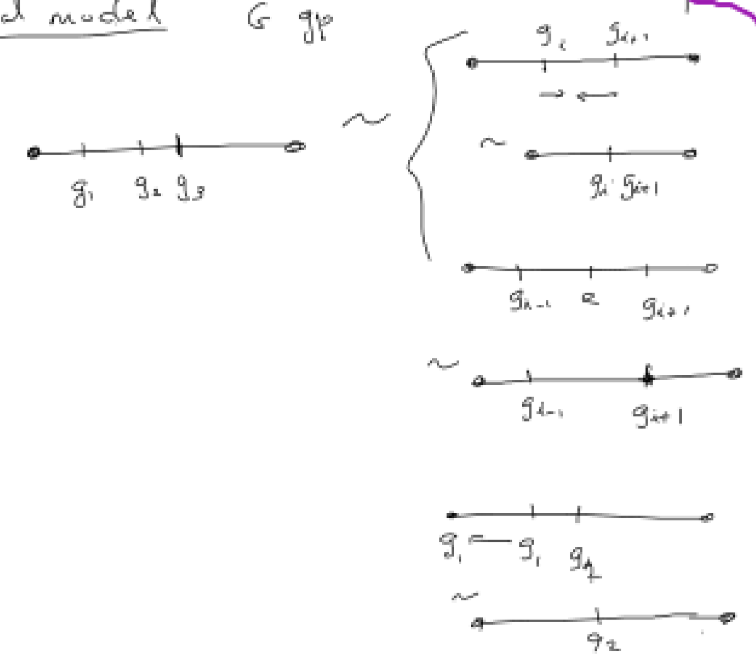
simplicial model (nerve)

"natural" geometric models

"classifying space"

<u>simplicial model</u> G gp

$B_\Delta G =$



GO TO ...

Formally.

$B_\Delta G = \bigsqcup \Delta^n \times G^n / \sim$

$(0 \le t_1 \le t_2 \le \dots \le t_n \le 1) \times (g_1 \dots g_n)$

s.t. $t_i = t_{i+1}$

$\sim (0 \le t_1 \le t_2 \le \dots \le t_i \le t_{i+2} \le \dots) \times (g_1 \dots g_{i-1}, g_i g_{i+1}, g_{i+2} \dots)$

$(0 \le t_1 \le \dots < t_{i-1}, t_i \le \dots) \times (g_1, g_2 \dots g_i = e, g_{i+1} \dots)$

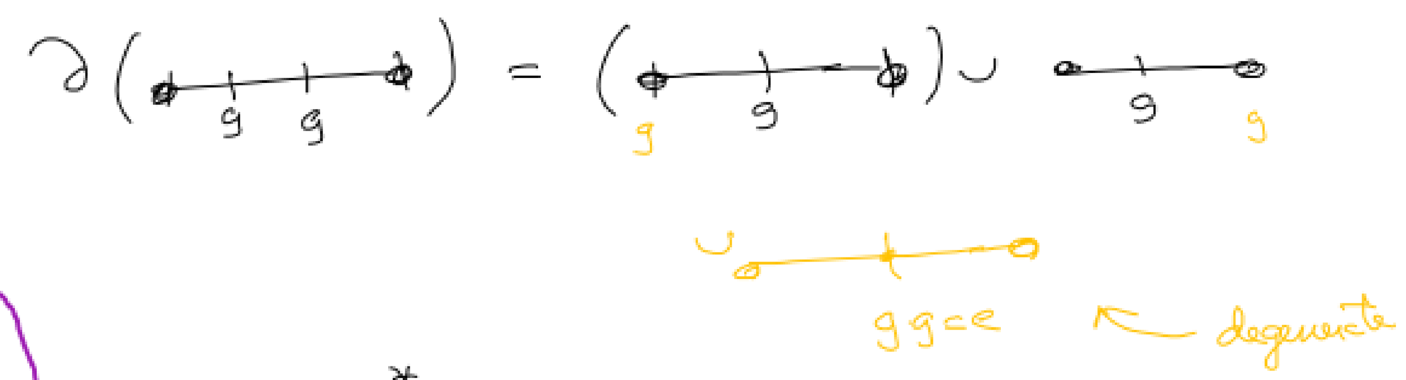
$\sim (0 \le t_1 \dots t_{i-1} \le t_{i+1} \le \dots) \times (g_1 \dots g_{i-1}, g_{i+1} \dots)$

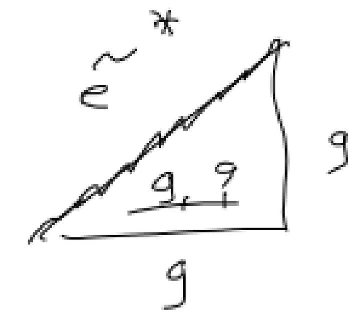
$(0 = t_1 \le t_2 \dots) \times (g_1, g_2, \dots)$

$\sim (t_2 \le t_3 \le \dots \le t_n) \times (g_2, g_3 \dots)$

②

<u>Ex</u> $G = C_2$ $\exists!$ n-cell. after these identifications





2-skel $\cong \mathbb{R}P^2$

n-skel $\simeq \mathbb{R}P^n$

$B_\Delta C_2 \simeq \mathbb{R}P^\infty$

Why $B_\Delta G$?

w/ mild hypotheses on X.

$[X, B_\Delta G] \longleftrightarrow$ principal G-bundles$/_X$

= locally trivial <u>free G-spaces</u> P

s.t. $P \longrightarrow P/_G \cong X$.

Homotopy characterization: If

$EG \longrightarrow BG$ is any principal G-bundle w/ $EG \simeq *$

• Then $[X, BG]$ classifies principal G-bdles

$\Rightarrow$ Any other $E'G \to B'G$ w/ $E'G \simeq *$.

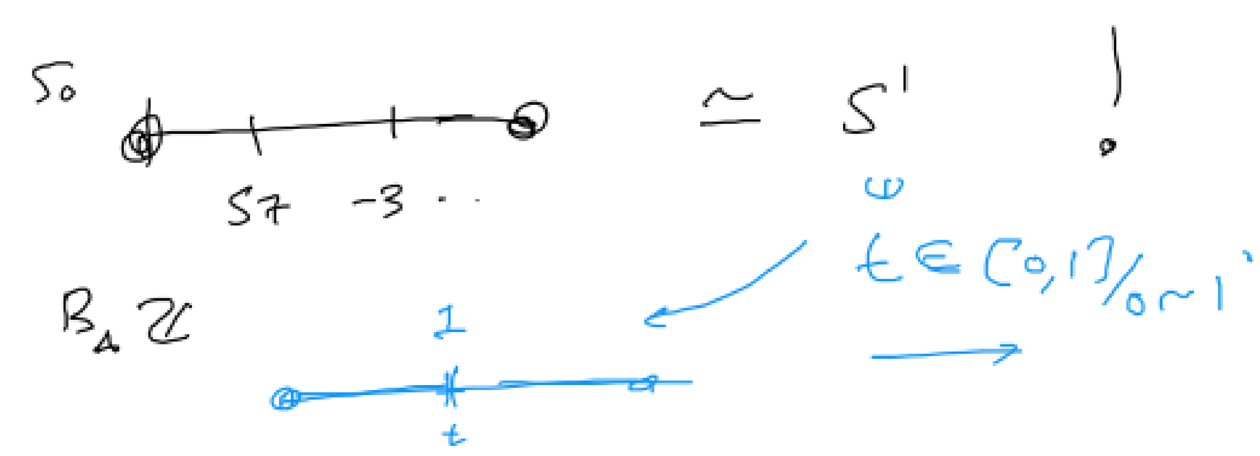
yields a homotopy equiv BG.

<u>Ex</u> $G = C_2$ $C_2 \circlearrowright S^\infty$ $x \longleftrightarrow -x$.

$S^\infty \simeq *$. $\Rightarrow BC_2 \simeq S^\infty/_{C_2} = \mathbb{R}P^\infty$.

$G = \mathbb{Z}$ $EG = \mathbb{R}$ $BG \simeq S^1$

So



"homework" direct geom proof?

Σ surface

<u>Ex</u> $\pi_1 \Sigma \circlearrowright (\tilde{\Sigma} \cong \mathbb{R}^2)$

$\Rightarrow \Sigma \simeq B(\pi_1 \Sigma)$

<u>Ex</u> $S^1 \circlearrowright \mathbb{C}^n$ $S^\infty \simeq *$ $\Rightarrow$ $S^\infty/_{S^1} \simeq BS^1$

$\cup\!\shortmid$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\shortparallel$

S^{2n-1} $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\mathbb{C}P^\infty$.