③

<u>Sketch</u> of proof of classification thm & homotopy characterization (uniqueness)

$$P \to X / \cong \;\longleftrightarrow\; [P, EG]^G/\text{htpy} \;\longleftrightarrow\; [P/G, EG/G] = [X, BG]/\text{htpy}$$

Suppose have EG, $E'G$

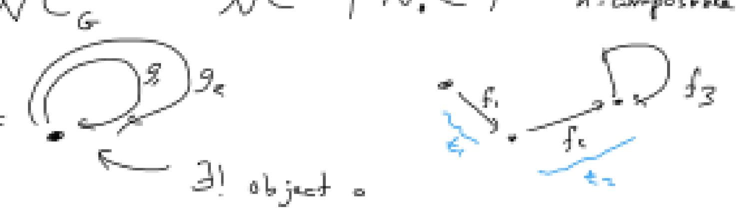
$$\begin{array}{ccccc} G & \longrightarrow & EG & \longrightarrow & BG \\ \| \simeq & & \uparrow \simeq & & \uparrow \simeq \\ G & \longrightarrow & EG \times E'G & \longrightarrow & (EG \times E'G)/G \\ \| \simeq & & \downarrow \simeq & & \downarrow \simeq \\ G & \longrightarrow & E'G & \longrightarrow & B'G \end{array}$$

In fact $\pi_{i+1}(BG) \cong \pi_i(G)$

($\pi_0 BG = *$)

Why is $B_\Delta G \simeq BG$?

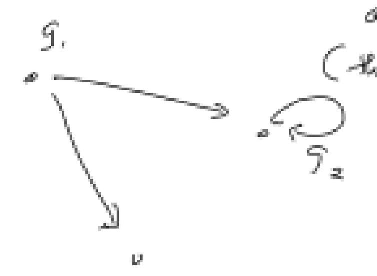
$B_\Delta G = \mathcal{N}\mathcal{C}_G$ $\qquad \mathcal{N}\mathcal{C} = |N_\bullet \mathcal{C}|$ n-simplices are n-composable morph.

$\mathcal{C}_G =$ 

$E_\Delta G = \mathcal{N}\tilde{\mathcal{C}}_G \qquad$ ob $= G$

∃! morphism between any two objects

(think $g_j \cdot g_i^{-1}$)



$$\mathcal{N}\tilde{\mathcal{C}}_G / G = \mathcal{N}\mathcal{C}_G$$

$\simeq *$ b/c any collection of simplices $\subseteq \Delta^D \subseteq \mathcal{N}\tilde{\mathcal{C}}_G \Rightarrow$ acyclic.

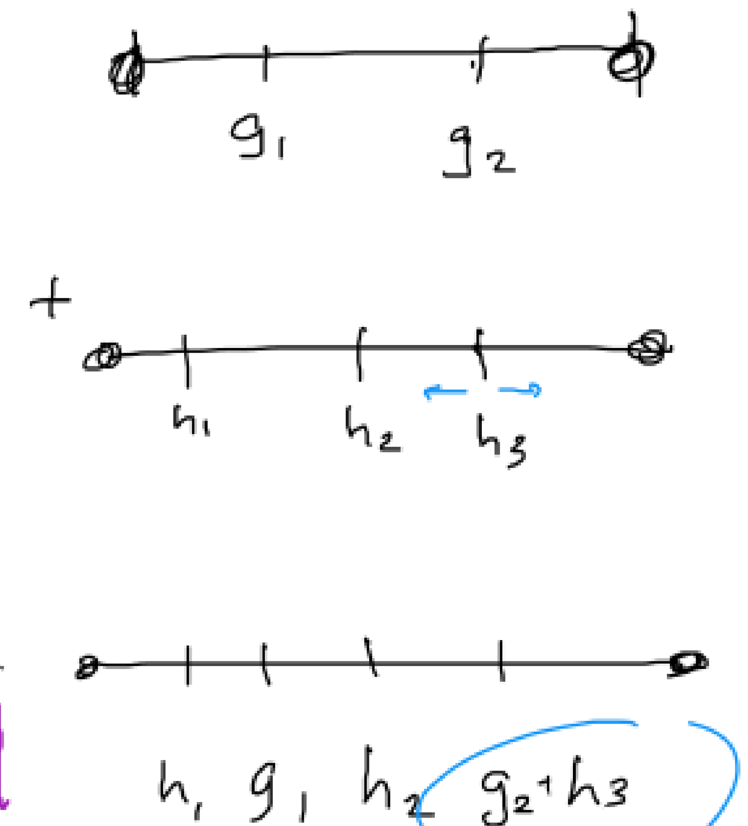
$C_*^{cw}(E_\Delta G; R) \cong$ resolution of R over $R[G]$

$G \longrightarrow BG$ "rich" <u>functor</u> gp $\longrightarrow$ top

immediate by $B_\Delta G$.

④

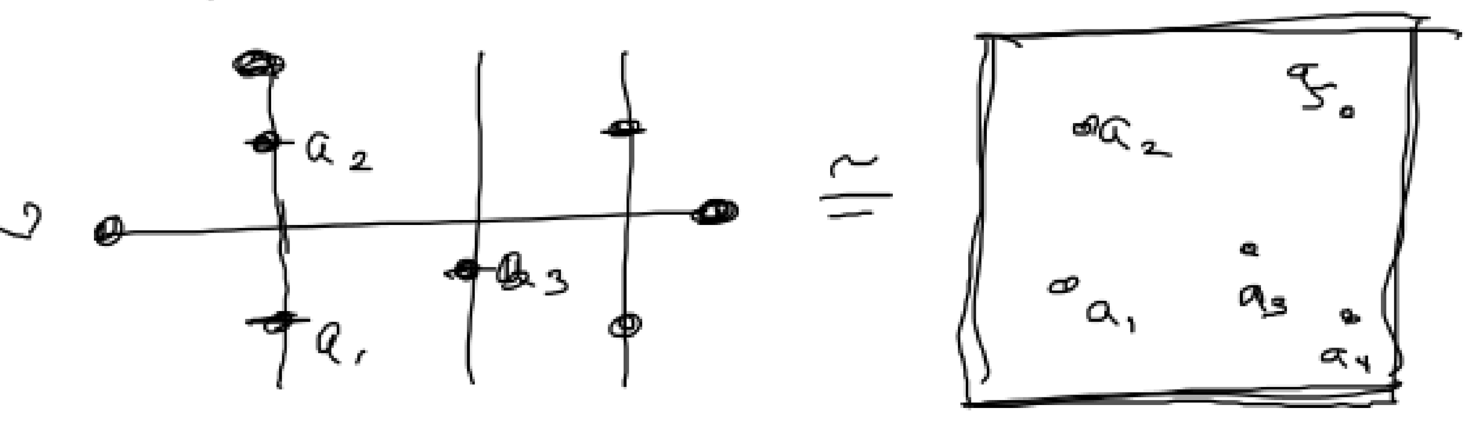
Propn: A abelian (top gp) then $B_\Delta A$ is an ab. top. gp.



need abelian.

Make BBA, $BBBA, \dots$!

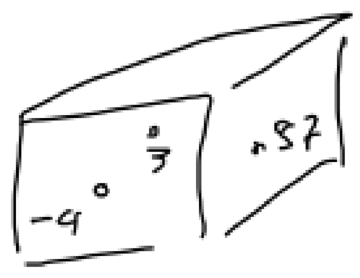
$A =$ fin generated ($\mathbb{Z}/n$)



<u>Ex</u> $A = \mathbb{Z} \qquad BB\mathbb{Z} \cong \mathbb{Z}S^2$

<u>Propn</u> Let A be f.g abelian gp. then

$B^n A$ is a $K(A, n)$



$K(\mathbb{Z}, 3)$ $\qquad \pi_{i+1}(BG) \cong \pi_i(G)$

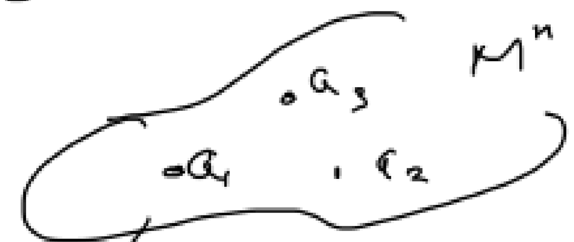
Later: some geometry of EM spaces.

Two generalizations

$$X \subseteq \mathbb{R}^n \subseteq \mathbb{R}^\infty$$

$\nu X \simeq X$. Instead of $\mathbb{Z}X$

take $\mathbb{S}X$ = free "sphere spectrum" (E_∞-alg) ob."

Factorization homology 

E_n-alg - elts.