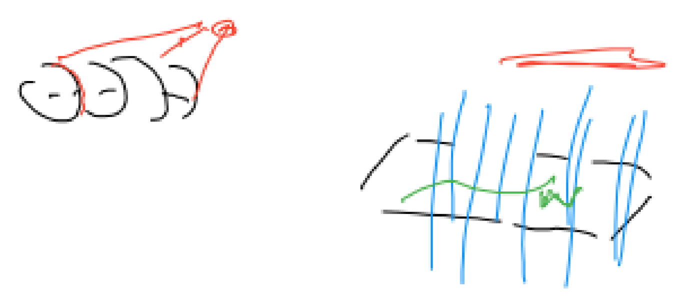
$\begin{matrix} E \\ \downarrow p \\ M \end{matrix}$ $z(M) \subseteq E$. $e(E) := \tau_{z(M)} \in H^{\cdot}(E; \mathbb{Z}$ or $\mathbb{Z}/2)$
zero section

To be in $H^{\cdot}(M)$, need $f: M \xrightarrow{\simeq} E$ in particular a section, $f \pitchfork z(M)$
$e(E) = \tau_{f^{-1}(z(M))}$.

Cor $e(E) = 0$ if E has a non-zero section

Close cousin of Thom classes in Thom $\cong$.

$Th(E) = D(E)/_{S(E)}$



M.

$\tau_{z(M)}$ Thom class

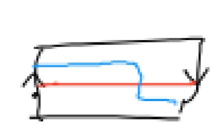
Thom $\cong$ $\tau_W \longrightarrow \tau_{z(M)} \in C^{\cdot}(Th(E))$
$W \subseteq M$ codim increases by rank E.

Other submflds constructed in E?

$w_n \longleftrightarrow \quad \subseteq E$ 1 vector lin. dep.
$w_{n-1} \longleftrightarrow \tau_{\cdot} \hookrightarrow \subseteq E \oplus E$ 2 vectors which are lin. dep. in E.
$\vdots$
$w_{n-i} \longleftrightarrow \tau_{\subseteq \underbrace{E \oplus \cdots \oplus E}_{i+1}}$ $i+1$ vectors are lin dependent.

Axioms/Properties.

- Functorial ✓
- values on $\varepsilon = M \times \mathbb{R}$ ✓
  Möbius bundle.



S^1 gen of H^1 ✓

- Whitney sum.
  $w_n(E \oplus F) = \sum_{i+j=n} w_i(E) \cdot w_j(F)$.

Two cases of new(?) proof.

E rank i, F rank j $i+j=n$.

$0 \oplus x$, $E \oplus F \supseteq y \oplus 0$, $w_i(E)$ E, F, $w_j(F)$

$\{0 \oplus x\} \cap \{y \oplus 0\} = \{0 \oplus 0\} \leftrightarrow w_n(E \oplus F)$
$w_i(E) \cdot w_j(F)$ $=$

- E, F line bundles. To show that
  $w_1(E \oplus F) = w_1(E)\, w_0(F) + w_0(E)\, w_1(F)$

pairs of lin. dep vectors in $(E \oplus F) \oplus (E \oplus F)$

$=$ (locally) $a\vec{e}_1 \oplus b\vec{f}_1 \oplus c\vec{e}_1 \oplus d\vec{f}_1$.
s.t. $ad = bc$.

$ad = s \cdot bc$
$s \in [0,1]$ $\longrightarrow$ $ad = 0$.

$a\vec{e}_1 \oplus b\vec{f}_1 + c\vec{e}_2 + d\vec{f}_1$
s.t. $a = 0$ or $d = 0$
$w_1(E) \cdot w_0(F)$ $w_0(E)\, w_1(F)$

- Has this occurred elsewhere?
- general proof?
- roles of coboundaries?

$\mathrm{Vect}^{\mathbb{C}}_n(X) \longleftrightarrow [X, BU(n)]$

- principal $U(n)$-bundle $\begin{matrix} Y \\ \downarrow \\ X \end{matrix}$ $\longleftrightarrow$ $Y \times_{U(n)} \mathbb{C}^n$ vector bundle

$\begin{matrix} \mathrm{Aut}(E) \\ \downarrow \\ X \end{matrix} \longleftarrow \begin{matrix} E \\ \downarrow \\ X \end{matrix}$

- $Gr_n(\mathbb{C}^\infty)$ is "natural" geometric model for $BU(n)$

$S^{2(N-n)-1}$
$\downarrow$
$U(n) \hookrightarrow$ n-frames in $\mathbb{C}^N$ $\xrightarrow{\text{quotient}}$ n-dim'l subspace of $\mathbb{C}^\infty$.
$\downarrow$
$(n-1)$ frames in $\mathbb{C}^N$

Digression mix simpl & nat'l geometric models

(Bruno Harris)