Char classes $\bigoplus_n H^*(BU(n))$ } pairing ?

Consider $\bigoplus H_*(BU(n))$

$= H_*(\coprod_n BU(n))$ ← homotopy-comm. H-space under $\oplus$ $\Rightarrow H_*$ bialgebra

$H_*(BU(1)) = H_*(\mathbb{CP}^\infty)$ spanned by $b_i = [\mathbb{CP}^i]$

$\Delta b_n = \sum_{i+j=n} b_i \otimes b_j$ (dual to $x^i \cup x^j = x^n$ $\forall\, i+j=n$)

What is pairing ?

<u>Ex</u>

	c_1^3	c_1c_2	c_3
b_1^{*3}	6	3	1
$b_2 * b_1 * b_0$	3	1	0
$b_3 * b_0^2$	1	0	0

Euler class

$\langle c_1^3, b_3 \rangle = 1$ in $H^*(\mathbb{CP}^\infty)$

$\langle c_1^3, b_1^3 \rangle$
$= \langle \mu^2(c_1\otimes c_1 \otimes c_1), b_1^3 \rangle = \langle c_1 \otimes c_1 \otimes c_1, \Delta^2 b_1^{*3} \rangle$
$= \langle c_1 \otimes c_1 \otimes c_1, (b_1 \otimes 1 + 1 \otimes b_1)^{*3} \rangle$
$= 6$

Compare with symmetric functions on $x_1, \ldots$

σ_i i^{th} elem. symm. fn.

	σ_1^3	$\sigma_1\sigma_2$	σ_3
$x_1x_2x_3^*$	6		
$x_1^2x_2 + x_1^2x_3 + \cdots$	3		
$x_1^3 + x_2^3 + x_3^3$	1		

$(x_1 + x_2 + x_3)^3$

- $H_*\, BU(n)$ is spanned $[\prod \mathbb{CP}^i]$
- $H_* / H^*\, BU$ is iso to a self-dual Hopf algebra of symmetric polynomials. Polynomial on σ_i, (c_i, b_i -) $\Delta \sigma_n = \sum \sigma_i \otimes \sigma_j$

Basis distinct from H_* / H^* is Schubert cells (in H_* through all str. H^* through Thom classes)

(D. Yang ?) How do these compare ?

Other bases ?

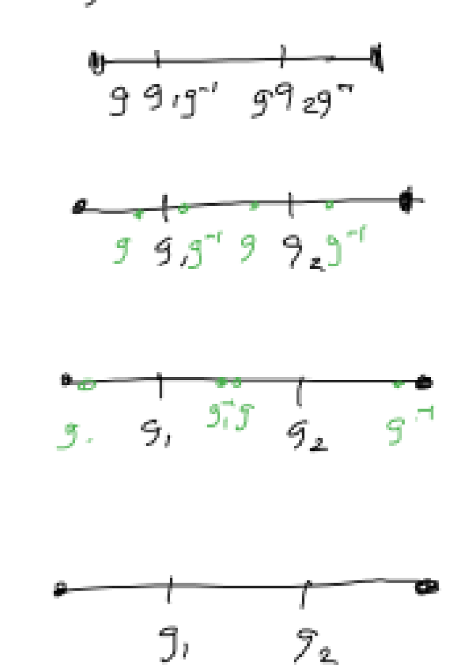
$\prod BU(1) \longrightarrow BU(n)$

$H^*(\prod BU(1)) \longleftarrow H^*\, BU(n)$ is injective.

$H \le G \Rightarrow BH \longrightarrow BG$ h_1 h_2

$\xleftarrow{\text{res}}$ H^*

By conjugation $g \circ G \curvearrowright BG$. Claim $\simeq$ id on BG.



$H \le G \Rightarrow N(H) \curvearrowright BH$.

restrict $H^*(BG) \longrightarrow H^*(BH)$

$H^*(BG) \to H^*(BH)^{NH/H = WH} \to H^*(BH)$

<u>Ex</u> $U(1)^n \le U(n)$ $- W = S_n$
$= H$ $= G$