★ Announcements:

- Lecture this Thursday - Hopf invts for π_1 (of $\vee S^1$).
- Harris paper on Bott periodicity will be posted.

Erratum (thanks Andrea Bianchi)

$E \oplus E \supseteq \{v_1, v_2 \text{ linearly dep}\}$ is not smooth mfld.
Δ is a cone on the locus on unit sphere to $\vec{0} \oplus \vec{0}$.

Fixes (possible)

- pass to sphere bundle ($\neq$ base. ? section??)
- "work around" singularities. codim 2 or greater.

$$f: \Delta^{n-1} \to E \oplus E \quad \begin{cases} f \pitchfork \Delta \\ f \pitchfork (\vec{0}\oplus\vec{0}) \Rightarrow f^{-1}(\vec{0}\oplus\vec{0} = \emptyset) \end{cases}$$

need for $H: \Delta^n \to E \oplus E$. $\delta \ldots$

- Define Thom cochains for proper co-oriented maps (extending from for immersions) - follow Quillen's "Elementary proofs of ..."
- Nir's suggestion: replace $E \oplus E$ by $\Lambda^2 E$. and play same game w/ 0 section codim ???

Thom cochain dictionary

submflds	—	cochains
∂	—	δ
$\cup$	—	$+$
$f^{-1}(\ )$	—	$f^{\#}$
$f(\ )$ proper $\Rightarrow \Sigma$ isomorphisms Thom domains.	—	$f_!$
$\cap \quad \pitchfork$	—	cup product ($+\delta$)
char classes normal bundle	—	Steenrod operations !

Wu formula: $W \subseteq M$ submfld. ν normal bundle.

$$f_! w_i(\nu) = Sq^i \tau_W.$$

★

E-M spaces. $B^n \mathbb{Z}/2 \simeq K(\mathbb{Z}/2, n)$ 

"Recall" $\tilde{H}^*(X; \mathbb{Z}/2) \cong [X, K(\mathbb{Z}/2, n)]$

$$f^*(\iota_n) \longleftarrow f$$

The latter satisfies axioms for $\tilde{H}^*$. Especially suspension $\cong$:

$$[\Sigma X, K(\mathbb{Z}/2, n+1)] \cong [X, \Omega K(\mathbb{Z}/2, n+1)]$$
$$\cong [X, K(\mathbb{Z}/2, n)]$$

as in general $\Omega BG \simeq G$.

$$\begin{array}{ccccc} \Omega BG & \to & PBG & \to & BG \\ \| & & \uparrow & & \\ \text{fib} & \to & PEG & \to & BG \\ \| & & \downarrow & & \\ G & \to & EG & \to & BG \end{array}$$

Rmk M monoid, BM exists. ΩBM is defined as group completion

$\Rightarrow [K(\mathbb{Z}/2, n), K(\mathbb{Z}/2, m)]$ can be composed w/ $[X, K(\mathbb{Z}/2, n)]$ to give H^* operations

akin to rep'n of GL_n act on $Rep(G)$

Thm $H^*(K(\mathbb{Z}/2, n); \mathbb{Z}/2)$ is poly. ring on classes $Sq^I(\iota_n)$ as I ranges over admissible seq's of excess $\le n$.

gen' of $H^*(K(\mathbb{Z}/2, n); \mathbb{Z}/2)$

LSSS argument of Borel.