Focus instead on $H$. Use cohomology structure.
(I don't understand everything as much as I'd like about "whole story" yet - $H$, $H'$ + all structure)

- Each $B^r \mathbb{C}[\mu]$ is a top $\mu$ group $\Rightarrow H$ is a cocommutative Hopf algebra.
- For $R$ a ring there is $B^r \mathbb{C}[\mu] \otimes B^r \mathbb{C}[\mu] \rightarrow B^{2r} \mathbb{C}[\mu]$
  \[
  \begin{array}{ccc}
  e & \otimes & e \\
  \mu & \mapsto & \begin{bmatrix}
    e \\
    \mu
  \end{bmatrix}
  \end{array}
  \]

There exists a $(\oplus (x, B^r \mathbb{C}))$ of structure of $\mathbb{C}$ ring

\[
  f: x \rightarrow B^r \mathbb{C}, \quad g: x \rightarrow B^r \mathbb{C}
\]

\[
  x \triangleright y = x \triangleright y \quad B^r \mathbb{C} \otimes \mathbb{C} \rightarrow \mathbb{C}
\]

Poincaré coincidence or cup product.

$\mathbb{P}^k \otimes H_k (B^r \mathbb{C}; \mathbb{P}_x)$ is a Hopf ring

Hypothesis:  $x_2, x_3 \in \text{Heis alg}$

Hypothesis:  $x_i$ an object in alg's

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Two products:  $x \otimes x$ and $\cdots$

\[
  a \otimes (a \otimes c) = \Delta (a) \otimes (a \otimes c)
\]

Thm (S. Wilson) $\otimes H_k (B^r \mathbb{C}; \mathbb{P}_x)$ is a

Hopf ring over $H_k (B^r \mathbb{C}; \mathbb{P}_x)$

Start with $H_k (B^r \mathbb{C}; \mathbb{P}_x)$

- dual to $H^2$. Polynomial gen'd alg.
  $\Rightarrow$ prim. ideal gen'd $\Rightarrow H$ is divided power
    $(\text{unless more})$

Geometrically $x \otimes$ multiplication is cellular

\[
  \overline{y_i} \in H_i (B^r \mathbb{C}) \begin{array}{ccc}
    1 & \otimes & \text{pt}
  \end{array}
\]

\[
  x_2 \otimes y_3 = (x_2 \otimes y_3) \otimes 1
\]

Divided powers.

$\otimes \mathbb{P}_x$. Exterior over $y_3$.

\[
  y_3 = (y_3 \otimes x_2 \otimes y_1)
\]

\[
  x \overset{\otimes 2} {\longrightarrow} y_1 \otimes y_1 \quad \text{dual to} \quad z_2 \in H^2
\]

rep'd by $S^2 \rightarrow B^2 \mathbb{P}_x$

\[
  \begin{array}{ccc}
    y_2 \otimes x \rightarrow (y_2 \otimes x) \quad \text{and claim}
  \end{array}
\]

\[
  y_1 \otimes x \otimes y_2
\]

\[
  y_1 \otimes x \otimes y_2 = (y_1 \otimes x \otimes y_2)
\]

\[
  \begin{array}{ccc}
    z \left( y_1 \otimes y_2 + 1 = y_1 \otimes y_2ight)
  \end{array}
\]

\[
  (1 \otimes y_2) \otimes (y_1 \otimes y_2)
\]

\[
  \begin{array}{ccc}
    \square \otimes \square = 0
  \end{array}
\]

\[
  y_1 \otimes x \otimes y_2
\]

\[
  \begin{array}{ccc}
    \begin{array}{ccc}
      y_1 \otimes x \otimes y_2
    \end{array}
  \end{array}
\]

Relationship to pairing $x$.

Standard $H'$ statement?

More elementary/geometric proof using $H$, $H'$ together?