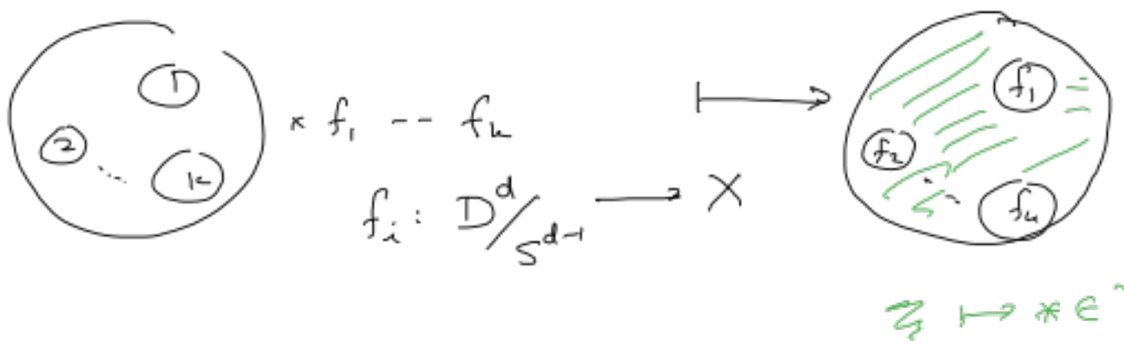
EX Disks^d acts on $\Omega^d X$.



Compare w/ proof that $\pi_2(X)$ is abelian.

$\Leftarrow$ the "space of choices" of mult on $\Omega^2 X$ is ctd.

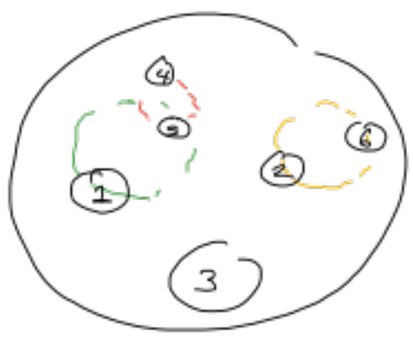
Apply $H_*(\ ; k)$ & Künneth.

Thm $H_*(\text{Disks}^d) \curvearrowright H_*(\Omega^d X)$.

$\| 2$

Pois^d

$\overset{ex}{\cong}$ $[x_1, [x_4, x_5]] \cdot [x_2, x_6] \cdot x_3 \longrightarrow$

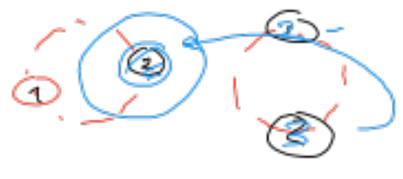


deg $3(d-1)$

Rmk Intuitively pretty clear. Operad insertion easy

$O(2) \times O(2) \longrightarrow O(3)$

$[x_1, x_2] \times [x_2, x_1] \xrightarrow{\sigma_2}$



Not as easy: $[x_1, x_2 * x_3] = \pm x_2 [x_1, x_3] \pm [x_1, x_2] \cdot x_3$

Easier to establish in H^* & use pairing.

So then an example of Central Thm 4)

$H_*(\Omega^n \Sigma^n(X); \mathbb{Q}) \cong$ Free gdd. Poisson alg on $\overline{H_*}(X)$

$H_*(\Omega^3(S^5 \vee S^6); \mathbb{Q})$

dg.	2	3		5		7	8	----
classes	x_2	y_3		xy		x^2y		
						$[x,y]$	$[y,y]$	

free comm. alg. on $\overline{H_*}(X)$
"answer" for $H_*(\Omega^\infty \Sigma^\infty X; \mathbb{Q})$

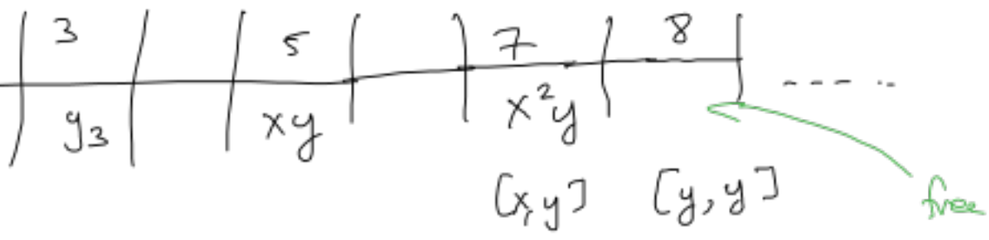
What is $*$, $[,]$?

$\sigma : W \longrightarrow \Omega^d X$ repres $[W], [V] \in H_*(\Omega^d X)$

$\tau : V \longrightarrow \Omega^d X$

$[W] * [V] =$ $W \times V \longrightarrow \Omega^d X \times \Omega^d X \xrightarrow{\mu} \Omega^d X$

$[[W], [V]] =$ $S^{d-1} \times W \times V \longrightarrow \text{Disk}^d(2) \times \Omega^d X \times \Omega^d X \longrightarrow \Omega^d X$.



Next time
- Fulton-MacPherson operad.
- Recognition Thm.
- Dyer-Lashof operations

$\rightsquigarrow$ H_* / H^* symm. gps.